Mathematics 5 — Quarter, Year

Final Examination

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Calculators, notes and books may not be used in this examination.

You may not receive full credit for a correct answer if insufficient work is shown, but you are not required to simplify your answers.

Only writing contained in the provided boxes will be scored.

The exam contains 24 questions with 208 possible points.

(144 points – Recall) Questions R1–R16 are each worth **nine points**.
(32 points – Analysis) Questions A1–A4 require work shown and are each worth **eight points**.
(32 points – Synthesis) Questions S1–S4 require work shown and are each worth **eight points**.

Signature

R1. Suppose that a line intersects the points $(1,3)$ and $(5,9)$. (i) Find an equation for the line. (ii) What is the y-intercept of the line?

R2. Suppose that f and g are given by

$$f(x) = 5x^2 + 1 \quad \text{and} \quad g(3) = 2.$$

Calculate $(f \circ g)(3)$.

R3. Find a parametric equation for a line L that has slope 2 and intersects $(5,7)$.

R4. Suppose that f is a degree two polynomial and g is a degree four polynomial. (i) What is the degree of $f \cdot g$? (ii) What is the degree of $f \circ g$?

R5. Find the position at time t of a particle moving at a constant velocity that is at $(1, 3)$ at time zero and $(2, 5)$ at time three.

R6. Line L_1 is given by the equation

$$y = 3x + 1.$$

Line L_2 is perpendicular to L_1 and intersects $(5, 1)$. Find an equation for L_2 written in slope/y-intercept form.

R7. Where is the vertex of the parabola given by $y = 2x^2 - 3x + 4$?

R8. How many degrees is $\frac{\pi}{5}$ radians?

R9. Line L_1 is given by the equation

$$y = x + 1$$

and L_2 is given by the equation

$$y = 2x - 3.$$

Where do L_1 and L_2 intersect?

R10. Write the function f given by

$$f(x) = |3x - 4|$$

as a piecewise defined function.

R11. One yard is three feet and one minute is sixty seconds. Write $5\,\frac{\text{feet}}{\text{second}}$ in units of yards and minutes.

R12. The graph of the invertible function f is a line with slope 3 that intersects $(1, 4)$. Find an equation for f^{-1}.

R13. Suppose that L is a line of slope 4 that passes through the origin. Where does L intersect the unit circle?

R14. Suppose that $\log_2(A) = 2$, $\log_2(B) = 3$, and $\log_2(C) = 5$. Calculate $\log_2\left(\frac{A^3C}{B^2}\right)$.

R15. Solve for x given that

$$4^{2x} \cdot \left(\frac{1}{2}\right)^{x+2} = 4^{3x+1}.$$

R16. A blob of a certain radioactive substance is decaying exponentially with respect to time. The mass is originally 20 grams. After 8 days, the mass is 5 grams. What is the half-life of the substance?

A1. Sketch the graph of the rational function f, where f is given by

$$f(x) = \frac{(x-4)^3(x-1)(x+3)^2}{(x-3)^2(x+2)}.$$

Use the graph to find all x so that $f(x) \leq 0$.

A2. Let p be the point $(2,5)$. Let L be the line that passes through $(1,2)$ and $(2,3)$. Find the point on L that is closest to p.

A3. A boat is initially at $(1,5)$ and travels at a constant velocity towards $(4,9)$. The boat travels at a speed of two miles per hour. Where will the boat be in 1.5 hours?

A4. Find all solutions of the equation $\sin^2(\theta) - \frac{3}{4} = 0$.

S1. The weight of paint is proportionate to its volume. Paint is always applied at the same thickness. It takes 8 ounces of paint to paint a ball that has a volume of 3 cubic feet. How many ounces of paint does it take to paint a ball that has a volume of 24 cubic feet?

S2. At time zero, Boat 1 is at $(2, 3)$ and Boat 2 is at $(9, 4)$. Boat 1 is traveling four miles per hour north (the positive y direction) and ten miles per hour east. Boat 2 is traveling two miles per hour north and one mile per hour east (the positive x direction). When are the boats closest to each other?

S3. Let f be the function defined by the formula

$$f(x) = x^3 + 2x.$$

What is an equation of the line tangent to the graph of f at the point $(1, 3)$? Be sure to use the algebraic definition of tangency—you will earn **no credit** for using other techniques.

S4. There is a building in front of you. The angle of elevation from your position to the top of the building is $16°$. You walk 50 feet towards the building and measure the angle of elevation to now be $37°$. How tall is the building?