Last Name, First Name	Discussion Section	Student ID

Worksheet 12 ● Tangency and Inverse Functions

1. Let f be the function defined by $f(x) = x^2$ on the positive real numbers. Find the equation of the line tangent to the graph of f at the point $(3, 9)$.

2. Graph the reflection of the graph of f and the line tangent to the graph of f at the point $(3, 9)$ about the line $y = x$.

3. The reflection of the graph of f about $y = x$ is the graph of a function, g. What function is it? The reflection of the line above is also a line. What line is it and what is its slope? Where do the reflected line and reflected graph meet?

If f is a rational function and L_1 is the line tangent to the graph of f at the point $(x, f(x))$ then define by $f'(x)$ the slope of L. If f^{-1} is the inverse function of f, then a line L_2 is tangent to the graph of f^{-1} at a point (a, b) if and only if the reflection of L_2 about the line $y = x$ is tangent to the graph of f at the point (b, a). In this case, let $(f^{-1})'(a)$ be the slope of L_2.

4. Calculate $(f^{-1})'(4)$ if f is the function given by $f(x) = x^3 + 2x + 1$. Note that $f(1)$ is equal to 4.

5. Repeat the above procedure but with the function f given by

$$f(x) = \frac{3x + 1}{x + 1},$$

but with the selected point on the graph being $(1, 2)$. After doing this, find the inverse of f and calculate the equation of the line tangent to the graph of f^{-1} at $(2, 1)$. Compare the equations of the two lines that you obtain. Carefully graph the two functions and the two lines as well as the line given by the locus of points $y = x$ to see if this makes sense.

6. Suppose that n is a natural number. Suppose that $g'(x) = x^{\frac{1}{n}}$. Use the above procedure to calculate $g'(a)$ for any non-zero a if n is odd and for strictly positive a if n is even. Why do we not attempt to calculate $g'(0)$?