

Homework One

Precalculus: Functions, Geometry, Trigonometry, & Modelling
UCR Math-005-E01, Summer 2020

Please upload your responses to these prompts to Gradescope before 8am next Monday morning. That way I have time to read over them to prepare for the coming week. There are three sections: Exercises, Writing, and Reflection. Please upload your responses to each section separately to help me keep organized.

gradescope.com/courses/125651

Exercises

Remember that these exercises are for you to practice. I want to see what you write down, like scratch work or your conclusions or whatever for each question, but don't waste a bunch of time meticulously writing details and stuff; I'm going to be spending much more reading over your Reflections and Writings than looking at what you write for these Exercises.

1. (NUMBER SENSE) Let a , b , and c be any real numbers. Suppose a and b are nonzero. Is it true that

$$\frac{c}{a} + \frac{c}{b} = \frac{c}{a+b}?$$

2. (NUMBER SENSE) Which of the two numbers 5^6 and 6^5 is bigger? What about the two numbers 11^{12} and 12^{11} ? For real numbers a and b , does $a \leq b$ always imply that $a^b \leq b^a$?

3. (NUMBER SENSE) Suppose you're in a precalculus class and you just took an exam. The exam was out of 100 points and had six questions, and your grade is determined by the percent of the points you earn. After the exam is graded and passed back, the instructor announces that one of the questions had an error in it, and so no one got any points on it. The question was worth 15 points. She then proposes a few options to change everyone's grades to make it fair:

A She could recalculate everyone's percent grade as if the exam were out of 85 points total instead of 100.

B She could simply award each student 15 extra points to make up for those points everyone missed.

C She could take all the other questions on the exam and uniformly make them each worth more points so that the exam is still out of 100 points. To do this she'd multiply the number of points each question is worth, and also the number of points you earned on each question, by $\frac{100}{85}$.

Which of these options gives you the highest percent score? Do any of these options actually result in the same percent score? Like, do any of these proposals give you exactly the same result even though they're described differently?

4. (ARITHMETIC TRICK) Can you rewrite the expression

$$\frac{x}{x-1}$$

in a way that only has a single x in it? HINT: Notice that $\frac{5}{4}$ and $\frac{37}{36}$ and $\frac{100}{99}$ are all examples of that expression after letting x be a

specific value. Notice how those fractions are all 1 *plus a little bit*. Can you rewrite that expression as 1 *plus a little bit*?

5. (DRILLS) Do you remember how to multiply and divide polynomials? If you want some practice, verify these computations.

$$(x^2 + 3)(x^3 - 7x^2 - x - 2) = x^5 - 7x^4 + 2x^3 - 23x^2 - 3x + 6$$

$$\left(\frac{x^2 - 3}{x^2 - 1}\right) + \left(\frac{x}{(x - 1)^2}\right) = \frac{x^3 - 2x + 3}{x^3 - x^2 - x + 1}$$

$$\frac{x^3 - 6x^2 + 11x - 6}{x - 2} = x^2 - 4x + 3$$

$$\frac{x^3 - 6x^2 + 11x - 6}{x - 1} = x^2 - 5x + 6$$

$$\frac{x^4 + 8x^3 + 7x^2 - 2x - 14}{x^2 + 2x + 2} = x^2 + 6x - 7$$

$$\frac{x^5 - 4x^2 + 6x - 24}{x^2 + 1} = (x^3 - x - 4) + \frac{7x - 20}{x^2 + 1}$$

Remember that you can Google terms like *polynomial division* or *adding rationals expressions* for more drills to practice.

6. (PARTIAL FRACTION DECOMPOSITION) Write each of the following rational expressions as a sum of rational expressions with indecomposable (unfactorable) denominators.

$$\frac{1}{x(x - 2)}$$

$$\frac{1}{(x^2 - x - 2)^2}$$

$$\frac{4x^2 - x - 1}{(x + 1)^2(x - 3)}$$

$$\frac{3x^3 + 7x^2 + 5x + 3}{(x + 1)^2(x^2 + 1)}$$

Remember you can check your calculations for these by adding together the rational expressions you get. And you can find more

examples and drills like this to practice by Googling *partial fraction decomposition*.

7. From the [University of Washington's Math120 book, page 28](#), work through the exercises

1.1 1.2 1.5 1.7 1.8 1.11 1.12 1.15

8. (SCALING ACROSS UNITS) Wine is draining out of a barrel at a rate of one cubic foot per minute. How fast is the wine draining in terms of cubic miles per decade?
9. (SCALING ACROSS UNITS) Suppose you're on the moon. You can clearly see out across the surface of the moon as far out as a mile, and, looking all around you, you count 11 craters. But then you find a small hill to stand on, and can now see clearly out up to two miles. About how many craters do you expect you can see from your hill? (Assume that craters are distributed uniformly over the moon's surface.)
10. (SCALING ACROSS UNITS) You have a box that has a volume of 100 cubic feet. It takes two people ten minutes to spray paint this box. How many minutes does it take three people to spray paint a similar box that has a volume of 500 cubic feet? (Assume that the area of surface to be covered is proportionate to both the number of people working and the time spent spray painting.)
11. (CLASSIC) Two trains are heading towards each other on the same set of tracks, each at 50 km/hr. At the moment the two trains are 200 km apart, a bee sitting on the front of one of the trains takes off flying towards the other train at 75 km/hr. It lands on the other train, and immediately takes off again headed back towards the

first train. The bee continues flying back and forth from train to train until the two trains collide. How much distance did the bee travel before the collision?

Writing

Whereas the previous Exercises are for you to practice what you've learned and for you to test yourself, these short writing assignments are (1) for you to get experience writing down your mathematical thoughts in words, which I suspect you haven't had much practice with in previous math classes, and (2) to give me something to read to assess your thinking: it's much easier to tell if you're thinking about things correctly from writing like this than from looking over calculations.

1. I'm going to show you an argument that, actually, $1 = 2$.

First let $x = y$. Since $x = y$ we have

$$x^2 = xy.$$

We can subtract y^2 from both sides of this equation, yielding

$$x^2 - y^2 = xy - y^2.$$

Then by factoring both sides of this equation, and dividing both sides by the common factor $(x - y)$, we get

$$\begin{aligned}x^2 - y^2 &= xy - y^2 \\ \implies (x + y)(x - y) &= y(x - y) \\ \implies x + y &= y.\end{aligned}$$

But remember that $x = y$, so the left-hand side of this equation is equal to $2y$. So we have

$$2y = y,$$

and cancelling the y from both sides we have $1 = 2$.

But we very well know that $1 \neq 2$. So there must be something horribly awry with the this argument. First identify where this argument goes wrong. Then imagine you need to explain to a classmate where this argument goes wrong. I want you to write down exactly what you'd say to this classmate to make sure they see what's wrong with this argument. Don't go overboard here; it shouldn't take more than a couple sentences to explain it.

2. The YouTube channel Numberphile has some cute mathematics-themed videos. Here's one about a better way to slice cake:

youtu.be/wBU9N35ZHIw

Explore some other Numberphile videos until you've learned something you think is interesting. For this writing assignment I want you *teach* this thing to someone. Imagine you're going to explain this thing you learned to someone you know, say a sibling or other family member or friend. Write out what you would say to them to teach them this thing. Draw out any math/calculations/diagrams that you would draw to show this person what you're talking about.

Reflection

I'm going to ask you to reflect on your own learning progress each week. This will be the first thing I read of your uploaded homework. Don't think of this reflection as a formal task; think of it more as a conversation starter with me. I'll ask you some specific questions that I've got on my mind, but generally just let me know how you're doing.

How are you?

Do you think you've understood the math for this homework? Are you satisfied with your responses to the Writing prompts?

Is there anything in particular you'd like me to look at more carefully in your Exercise and Writing responses? Like, is there anything in particular you'd like feedback on?

Also, some brief logistics questions: Do you look at the homework assignments on your phone or on a desktop? Do you download the homework from my website, or look at it on Gradescope? I made the PDF for this homework a size more suitable for phones and I want to know if this is better, or if it's unnecessary.