Homework Three

Precalculus: Functions, Geometry, Trigonometry, & Modelling

UCR Math-005-E01, Summer 2020

Please upload your responses to these prompts to Gradescope before 8am next Monday morning. That way I have time to read over them to prepare for the coming week. You will upload all three of these sections under a single submission on Gradescope. When uploading the three sections, please keep them in the order listed here as Reflection, then Writing, then Exercises. There are (terse) solutions to some exercises at the end of this document, and remember that the University of Washington book has solutions in the back on page 291.

`gradescope.com/courses/125651`

Reflection

I'm putting the Reflection section first because I want to read your reflection first, but this should certainly be the last thing you write on this homework.

Do you think you've understood the math for this homework? Please respond to these in detail; I hope to get a complete picture of how you're doing just by reading this reflection.

Is there anything you'd like me to look at more carefully in your Exercise and Writing responses? If there is, please indicate where it is in the rest of your submission. Like, circle it with a bright color or something.

Writing

1. (TEACHING A PEER) This is exercise 7.16 in the University of Washington textbook:

   > *Sven starts walking due south at* 5 *feet per second from a point* 120 *feet north of an intersection. At the same time Rudyard starts walking due east at* 4 *feet per second from a point* 150 *feet west of the intersection.*
   >
   > *(a) Write an expression for the distance between Sven and Rudyard t seconds after they started walking.*
   >
   > *(b) When are Sven and Rudyard closest? What is the minimum distance between them?*

   First, on your own figure out how to respond to this exercise. Then the purpose of this writing prompt is for you *teach* a peer how to answer these questions. Write out a response to these questions as if you're *teaching another student in this class* the mathematical concepts behind answering these questions.

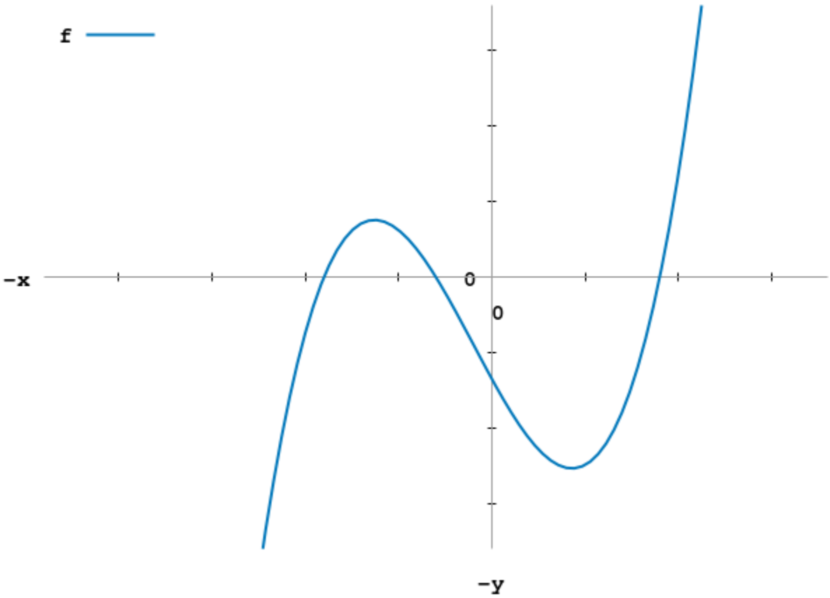
2. (TEACHING A CHILD) "*Subtracting a negative number is the same as adding that number*," is something we all know. For example we know that:

   $$3 - (-5) = 3 + 5$$

   But why is this true? Suppose you have a six-year-old cousin who just heard this fact in school, but doesn't understand why this is true. What would you say to them to help your cousin make sense of this? (Teaching children is hard. I want you to really think about this one.)

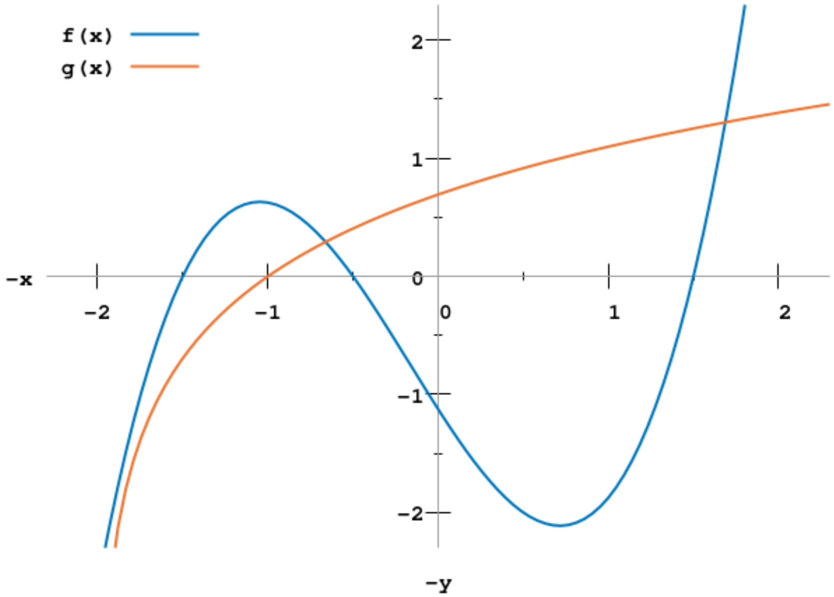
Exercises

1. Here is the graph of a function f.



Sketch a large copy of this graph, and then on the same sketch add the graphs of the functions $f+2$ and $2f$ and $1/f$.

2. Here are the graphs of two functions f and g.



Sketch the graphs $f+g$ and $g-f$ and fg and f/g and $g \circ f$ and g^{-1}. You'll certainly not want to do all this on the same set of axis.

3. We haven't talked about all of these functions yet, but you should know at some point what their graphs look like, and the prominent features of their graphs (x-intercepts, y-intercepts, asymptotes, domain, range, etc).

$\frac{1}{x}$	$\frac{1}{x^n}$ for integer n	$\lvert x \rvert$
e^x	$\ln(x)$	$\sqrt{x}$
$\sin(x)$	$\cos(x)$	$\tan(x)$

4. Graph these polynomial functions. What are the important features of such a graph? Make sure these features are noted on your graph. Make it clear to me you *know exactly* how to graph these. While I want you to know how to graph these manually (otherwise what's the point of this exercise?) but you can certainly use desmos to check your graph.

$$f(x) = \frac{1}{2}x - 3 \qquad g(x) = 2(x-3)(x+\pi)$$

$$h(x) = (x+1)^2 + 2 \qquad k(x) = x^2 + 11x + 30$$

$$\ell(x) = x^3 - x^2 - x + 1 \qquad m(x) = \mathrm{e}^{\pi}$$

5. Graph these rational functions. What are the important features of such a graph? Make sure these features are noted on your graph. Make it clear to me you *know exactly* how to graph these. While I want you to know how to graph these manually (otherwise what's the point of this exercise?) but you can certainly use desmos to check your graph.

$$p(x) = \frac{x^2 + x - 6}{x} \qquad q(x) = \frac{x^2 + 2x}{(x^2-1)^2}$$

$$r(x) = \frac{(x+3)^2(x+\mathrm{e})^5(x-2)(x-\pi^2)^2}{(x+\pi)^3(x+2)(x-1)^2}$$

6. Graph these piecewise-defined (multipart) functions. What are the important features of such a graph? Make sure these features are noted on your graph. Make it clear to me you *know exactly* how to graph these. While I want you to know how to graph these manually (otherwise what's the point of this exercise?) but you can certainly use desmos to check your graph. Also, note the implied domain of each of these functions.

$$f(x) = \begin{cases} x^3 & \text{for } x < 1 \\ 2 - x & \text{for } 1 \leq x \leq 3 \\ 5 - x & \text{for } 3 < x \end{cases}$$

$$g(x) = \begin{cases} \frac{1}{x} & \text{for } x < -\mathrm{e} \\ |x+1| + 1 & \text{for } -1 < x < 3 \\ 5 & \text{for } 3 < x \end{cases}$$

7. From the University of Washington's Math120 book, page 68, (PDF page 88) work through the exercises

   5.3 5.5 5.6 5.8

8. From the University of Washington's Math120 book, page 82, (PDF page 102) work through the exercises

   6.1 6.2 6.4 6.5 6.7 6.8 6.12

9. Tell me the maximum value of the function $f(x) = -x^2 + 2x + 4$. (Don't use calculus, and don't use a graphing calculator. You can answer this question with some brisk arithmetic, by calculating a few values of the function, and thinking about the *symmetry* of a parabola.)

10. From the University of Washington's Math120 book, page 103, (PDF page 123) work through the exercises

    7.1 7.2 7.3 7.7

    7.9 7.10 7.11 7.12 7.13 7.14

    Some of the ones in the second row are exercises we talked about in class Thursday.

11. You have 400 ft of fencing, and you want to fence in a rectangular region in your backyard for a garden.

    (a) What should the side lengths of this rectangle be such that the fenced in area is maximal? What shape will the region be? (You might already have some intuition on this and *know* what shape the region will be, but for this exercise I challenge you to justify your intuition mathematically.)

    (b) Suppose now there is a stone wall in your backyard that you can use to enclose your garden, so you only need build three sides of fencing. What are the side-lengths of your fenced in region now?

12. Two ships are sailing on the open sea near a small communications buoy. One ship is one league east and four leagues north

of the buoy with a heading $\langle 3, 4\rangle$. The other ship is seven leagues east and two leagues north of the buoy with a heading $\langle 1, 9\rangle$.

(a) Impose a coordinate system and illustrate this situation.

(b) How far apart are the boats initially?

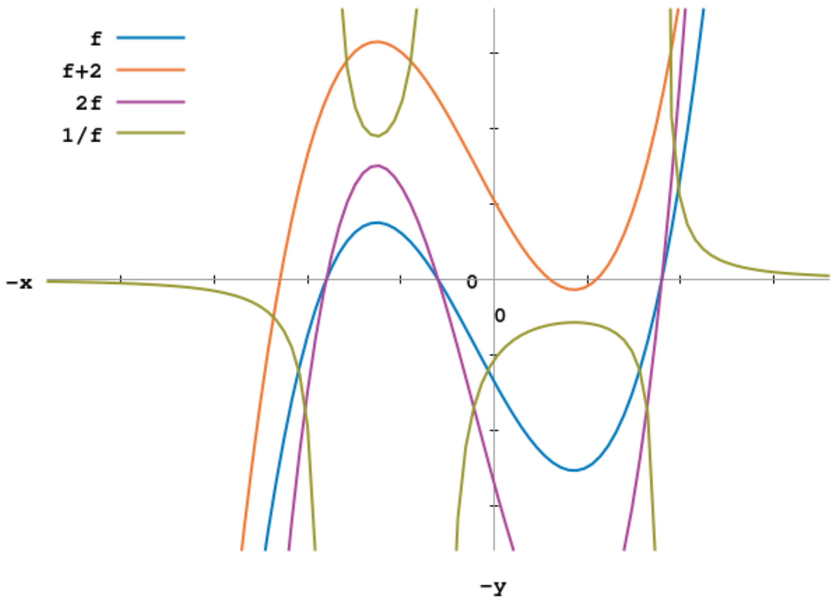
(c) Do the paths of the boats cross? And if they do, at what coordinates do they cross?

(d) If the boats don't collide, what's the closest the boats get to each other?

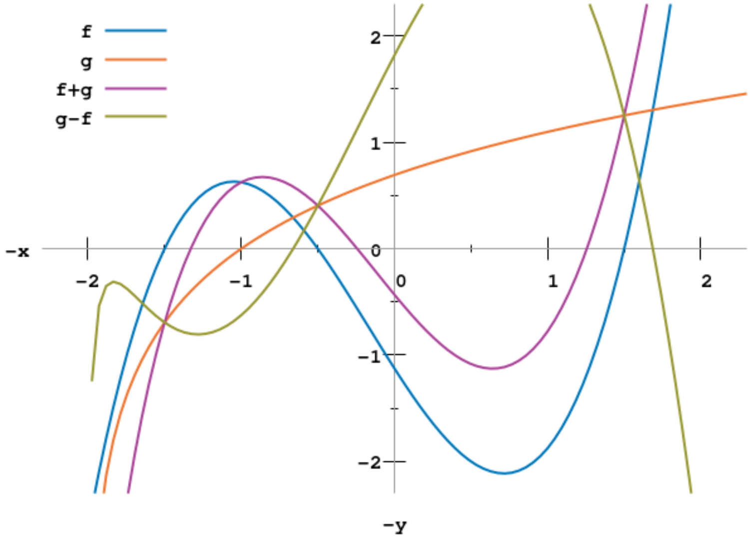
Some Solutions

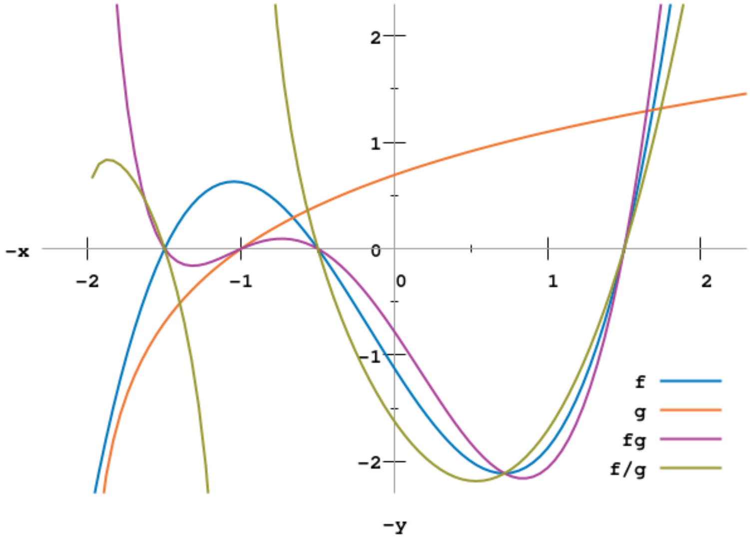
Note that there are solutions the University of Washington's book's exercises on page 291.

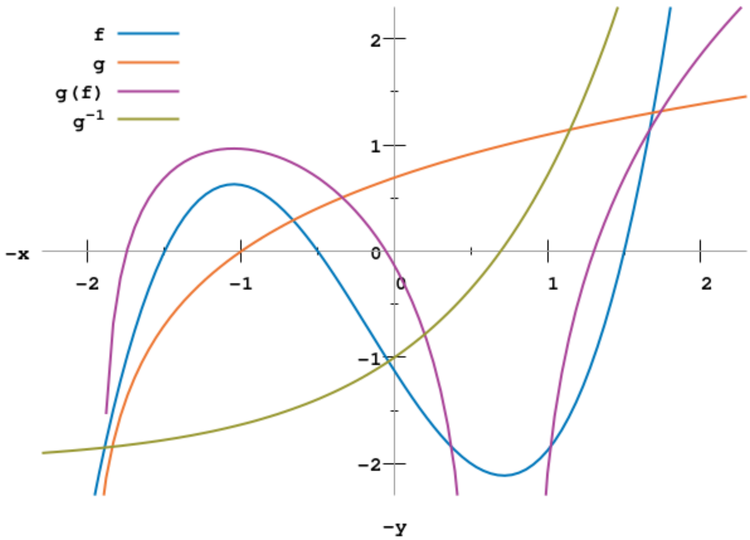
1 Here are the graphs for question one.



2 Here are the graphs for question two.







4 I think most of these are straightforward. For $\ell(x)$ you've got to recognize that either 1 or -1 is a root, and divide that root out. Then $m(x)$ looks goofy, but it's just a constant function.

9 Checking a few values of f, we see that $f(0) = 4$ and $f(1) = 5$ and $f(2) = 4$. Since $f(0) = f(2)$, the vertex must live at the x-value halfway between 0 and 2. So the max value is 5.

11 If your rectangle has side-lengths w and h, we need to maximize the function for area $A = wh$. But we know $2w+2h = 400$, so we're really maximizing $A(w) = w(200 - w)$, now a function of a single variable w. The roots of this are at $w = 0$ and $w = 200$, so the vertex occurs at $w = 100$.

If there is a stone wall in your back yard, you still need to maximize $A = wh$, but now you're constraint equation is $w + 2h = 400$.

12 Using the buoy as the origin, and aligning north with the positive y-axis and aligning east with the positive x-axis, the two boats' A and B positions are given by the functions

$$p_A(t) = (1,4) + t\langle 3,4\rangle \qquad p_B(t) = (7,2) + t\langle 1,9\rangle\,.$$

Written another way, as a function of t, boat A is at position $(1+3t, 4+4t)$ and boat B is at position $(7+t, 2+9t)$, And the distance between these points is given by the function $d(t)$ defined as

$$d(t) = \sqrt{(1+3t-7-t)^2 + (4+4t-2-9t)^2}\,.$$

To find the closest the boats get, you've got to minimize this function d, but since the $\sqrt{\ }$ function is strictly increasing, all you really need to do is find the minimum of the quadratic inside the radical to find the time t at which the boats are closest.