

Putnam Practice Set #7

1. Find the number  $R(n)$  of regions in which the plane can be divided by  $n$  straight lines.
2. Define a set to be **selfish** if it has its own cardinality (number of elements) as an element of itself. Find, with proof the number of subsets of  $\{1, 2, \dots, n\}$  that are minimal selfish sets; that is, subsets that are selfish and do not properly contain any other selfish set.

3. Evaluate

$$\sqrt[8]{2207 - \frac{1}{2207 - \frac{1}{2207 - \frac{1}{2207 - \dots}}}}.$$

4. Prove that if we paint every point of the plane in one of three colors, there will be two points one inch apart with the same color. Is this result necessarily true if we replace three by nine?
5. Call a set of positive integers **conspiratorial** if no three of them are pairwise relatively prime. What is the largest number of elements in any conspiratorial subset of integers 1 through 16?
6. Which is larger,  $2010^{2010}$  or  $2011^{2009}$ ?