Factorization in Polynomial Rings

Let R be a ring; $R[x_1, \ldots, x_n]$ the ring of polynomials in n variables.

- For polynomials $f, g \in R[x_1, \ldots, x_n]$, $\deg(f+g) \leq \deg(f) + \deg(g)$. Equality holds if neither leading coefficient of f or g is a zero divisor. In particular, if R is an integral domain, we always have $\deg(f+g) = \deg(f) + \deg(g)$.
- If R is a field, we can use a division algorithm in $R[x]$; i.e., if f, g are nonzero polynomials in $R[x]$, then there exist polynomials $q, r \in R[x]$ such that
$$f = qg + r, \quad \deg(r) < \deg(g).$$
- If R is a field, then $R[x]$ is a Euclidean domain. The units of $R[x]$ are the nonzero constant polynomials.
- Let R be a commutative ring with identity; $f \in R[x]$. Then $c \in R$ is a root of f if and only if $(x - c)$ divides f.