General Remarks on

Stable Homotopy Theory, Part V:
Duality and Thom Spectra

by J. Michael Boardman

The material posted in the accompanying file

http://math.ucr.edu/∼res/duality/Boardman-V.pdf

is the fifth part of notes by J. M. Boardman that were mimeographed at the University of Warwick in 1965–1966 and covered the material in his 1964 D. Phil. thesis, *On Stable Homotopy Theory and Some Applications*, written at Cambridge under C. T. C. Wall.

Parts I – IV of Boardman's notes contain his breakthrough work on constructing a mathematically sound version of stable homotopy theory which overcame the main objections to earlier attempts. Excellent summaries of this material, its impact, and more recent developments in the area are discussed in the proceedings of the 1998 conference held in honor of Professor Boardman's sixtieth birthday [4]. Descriptions of Boardman's stable homotopy category — or equivalent reformulations — appeared during the following decade, and the writeups by R. Vogt [5] and J. F. Adams [1] are particularly noteworthy examples. There has also been a great deal of work on refinements, variants and analogs of Boardman's work on stable homotopy categories, but all this is beyond the scope of the current discussion.

In addition to an introduction with simply titled "Stable Homotopy Theory" and Parts I – V which we have already mentioned, there was also a Part VI, which was superseded by a later article by Boardman on his solution to the Conner-Floyd **Five Halves Problem** for smooth involutions on closed manifolds [3].

On the other hand, even though the material in Part V has been used and cited repeatedly in the literature and has been extremely influential in the study of transfer or "wrong way" morphisms (compare [2]), detailed treatments of several key topics in Part V have not appeared in the literature in the four decades since Boardman wrote up his account. Therefore I am posting Chapter V of the original notes in order to make their contents more widely accessible. Although the discussion is formulated in terms of Boardman's stable homotopy category, aside from the explicit definitions of Thom spectra in Section 1 (pp. 5–7) the treatment is valid in virtually every standard framework for stable homotopy theory. I have also appended a list of references that are cited in the notes.

A Table of Contents for the notes and bibliographic citations for the preceding discussion appear on the next page.

Acknowledgment.   I am grateful to Professor Boardman for granting me permission to post his notes on my web site.

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References

[1] J. F. Adams. "Stable Homotopy and Generalised Homology" (notes by R. Ming), Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1974.

[2] J. C. Becker and D. H. Gottlieb. *A history of duality in algebraic topology.* Chapter 25 in "History of Topology" (I. M. James, *ed.*), pp. 725–745. North-Holland, Amsterdam, NL, 1999. — Modified version online: http://www.math.purdue.edu/~gottlieb/Papers/duality11.pdf

[3] J. M. Boardman, *Cobordism of involutions revisited.* "Proceedings of the Second Conference on Compact Transformation Groups (Univ. Massachusetts, Amherst, MA, 1971), Part I," [Springer] Lecture Notes in Math. Vol. 298, pp. 131–151, Springer-Verlag, Berlin-*etc.*, 1972.

[4] J. - P. Meyer, J. Morava and W. S. Wilson. "Homotopy invariant algebraic structures — A conference in honor of J. Michael Boardman. (Proc. AMS Special Session on Homotopy Theory, Baltimore, 1998)," Contemp. Math. Vol. 239. American Mathematical Society, Providence, RI, 1999.

[5] R. M. Vogt. "Boardman's stable homotopy category." [Aarhus] Lecture Note Series No. 12. Matematisk Insitut, Aarhus Universitet, Aarhus (DK), 1970.