## Computing the area of a circular sector

In the drawing below, the region colored in yellow is a circular sector whose angular size is $\boldsymbol{\theta}$, where $\boldsymbol{\theta}$ is acute (between $\mathbf{0}$ and $1 / 2 \pi$, measured in radians). The combined area of the regions colored in yellow and green is equal to

$$
\int_{0}^{\sin \theta} \sqrt{1-x^{2}} d x
$$

and the area of the triangular region colored in green is easy to compute; if we take the difference of these areas, we obtain the area of the circular sector colored in yellow. The definite integral and its indefinite counterpart can be found using a table of antiderivatives (indefinite integrals) or the standard integration techniques from single variable calculus.


We can use the same picture to find the area of a sector if the angle's measure is greater than $\boldsymbol{\pi} / \mathbf{2}$ radians by noting that in general $\boldsymbol{\theta}$ can be written as $\boldsymbol{k} \boldsymbol{\pi} / \mathbf{2}+\boldsymbol{\alpha}$, where $\boldsymbol{k}$ is an integer between $\mathbf{0}$ and $\mathbf{3}$, and $\boldsymbol{\alpha}$ is between $\mathbf{0}$ and $\boldsymbol{\pi} / \mathbf{2}$. If $\boldsymbol{k}$ is positive, then such a sector is given by taking the union of the region shaded in yellow with $\boldsymbol{k}$ of the adjacent blue quarter circles going counterclockwise from the left hand edge of the yellow region.

