In this discussion we shall view the real projective plane as the quotient space of the 2–disk by
the equivalence relation which identifies diametrically opposite points on the boundary. The
drawing above represents a minimal triangulation of the real projective plane; the colored
arrows represent pairs of edges that are identified under the equivalence relation.

If we rename the vertices 1, 2, 3, 4, 5, 6 as A, B, C, D, E, F respectively, then twice the 1–
cycle \( AB + BC – AC \) is the boundary of the following 2–chain in the standard chain complex
for the given triangulation of the real projective plane with the alphabetical ordering of its
vertices:

\[
ABD + BCD – ACE + ABF + BCF – ACD + DEF – BDE – AEF + CDF
\]

However, there is no 2–chain whose boundary equals \( AB + BC – AC \). It is possible to prove
this directly from the definition of the boundary map \( d_2 \) from 2–chains to 1–chains, but later in
the course we shall give a more conceptual and less computational proof in the exercises.