

General Laws

If L , M , c , and k are real numbers and

$$\lim_{x \rightarrow c} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = M, \quad \text{then}$$

Sum Rule: $\lim_{x \rightarrow c} (f(x) + g(x)) = L + M$

Difference Rule: $\lim_{x \rightarrow c} (f(x) - g(x)) = L - M$

Product Rule: $\lim_{x \rightarrow c} (f(x) \cdot g(x)) = L \cdot M$

Constant Multiple Rule: $\lim_{x \rightarrow c} (k \cdot f(x)) = k \cdot L$

Quotient Rule: $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0$

The Sandwich Theorem

If $g(x) \leq f(x) \leq h(x)$ in an open interval containing c , except possibly at $x = c$, and if

$$\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} h(x) = L,$$

then $\lim_{x \rightarrow c} f(x) = L$.

Inequalities

If $f(x) \leq g(x)$ in an open interval containing c , except possibly at $x = c$, and both limits exist, then

$$\lim_{x \rightarrow c} f(x) \leq \lim_{x \rightarrow c} g(x).$$

Continuity

If g is continuous at L and $\lim_{x \rightarrow c} f(x) = L$, then

$$\lim_{x \rightarrow c} g(f(x)) = g(L).$$

Parametric Equations

If $x = f(t)$ and $y = g(t)$ are differentiable, then

$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{d^2y/dt^2}{dx/dt}$$

General Formulas

Assume u and v are differentiable functions of x .

Constant: $\frac{d}{dx}(c) = 0$

Sum: $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Difference: $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

Constant Multiple: $\frac{d}{dx}(cu) = c \frac{du}{dx}$

Product: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

Quotient: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Power: $\frac{d}{dx}x^n = nx^{n-1}$

Chain Rule: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x \quad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Specific Formulas

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$, then

$$\lim_{x \rightarrow c} P(x) = P(c) = a_n c^n + a_{n-1} c^{n-1} + \dots + a_0.$$

If $P(x)$ and $Q(x)$ are polynomials and $Q(c) \neq 0$, then

$$\lim_{x \rightarrow c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}.$$

If $f(x)$ is continuous at $x = c$, then

$$\lim_{x \rightarrow c} f(x) = f(c).$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$