

Syllabus. Deformation theory

- Deformation quantization of smooth manifolds, star-products, Poisson structures. Examples: Lie algebras, symplectic manifolds.
- Hochschild cohomology of an associative algebra, algebraic structures on Hochschild cochain complex.
- Differential graded Lie algebras, Maurer-Cartan elements, Deligne's 2-groupoid.
- Quasi-isomorphisms of differential graded Lie algebras. The Goldman-Millson theorem. Canonical free resolution of a differential graded Lie algebra.
- Kontsevich's formality theorem for \mathbb{R}^n .
- Fedosov's resolutions, extension of Kontsevich's formality theorem to an arbitrary manifold.
- Applications. Poisson manifolds. Star-products. Poisson cohomology versus Hochschild cohomology.

The course is based on the following original articles:

References

- [1] F. Bayen, M. Flato, C. Fronsdal, A. Lichnerowicz, and D. Sternheimer. Deformation theory and quantization. I. Deformations of symplectic structures. *Ann. Phys. (N.Y.)*, **111** (1978) 61;
Deformation theory and quantization. II. Physical applications. *Ann. Phys. (N.Y.)*, **110** (1978) 111.
- [2] F.A. Berezin. Quantization. *Izv. Akad. Nauk.*, **38** (1974) 1116-1175;
General concept of quantization. *Commun. Math. Phys.*, **40** (1975) 153-174.
- [3] V. A. Dolgushev, Covariant and equivariant formality theorems, *Adv. Math.*, **191**, 1 (2005) 147-177; math.QA/0307212.
- [4] E. Getzler, A Darboux theorem for Hamiltonian operators in the formal calculus of variations, *Duke Math. J.* **111**, 3 (2002) 535-560.
- [5] W. Goldman and J. Millson, The deformation theory of representation of fundamental groups in compact Kähler manifolds, *Publ. Math. I.H.E.S.*, **67** (1988) 43-96.

- [6] M. Kontsevich, Deformation quantization of Poisson manifolds, *Lett. Math. Phys.*, **66** (2003) 157-216; q-alg/9709040.
- [7] D. Quillen, Rational homotopy theory, *Annals of Math.*, **90**, 2 (1969) 205–295.

Final project.

I will list 5-7 original research papers on deformation quantization. Each student will have to choose one of the papers read it and write a one-page review. This review should describe the results of the paper and what the student learnt from this paper. Several students may choose the same paper. In fact all students may choose the same paper. But they have to write the reviews independently.