Syllabus.

Differential Geometry and Topology, 440-3.

Suggested text books: M. Do Carmo, *Riemannian geometry*; P. Griffiths and J. Harris, *Principals of Algebraic geometry*; R. Bott and L.W. Tu, *Differential forms in algebraic topology*; Sh. Kobayashi and K. Nomizu, *Foundations of differential geometry*; R.O. Wells, *Differential analysis on complex manifolds*.

Homework assignments will be posted on the blackboard. They will consist of exercises from the lectures.

On May 25 (Friday) I will hand out **the take home final exam**. It will be due on June 1.

- **Rudiments of differential topology**
  1. De Rham complex on $\mathbb{R}^n$
  2. Meier-Vietoris sequence and applications
- **Complex manifolds**
  1. Almost complex manifolds, complex manifolds
  2. Calculus on complex manifolds
- **Sheaves and cohomology**
  1. Presheaves and sheaves
  2. Resolutions of sheaves
  3. Cohomology theory
- **Elliptic operator theory**
  1. Sobolev spaces
  2. Differential operators
  3. Pseudodifferential operators
  4. A parametrix for an elliptic differential operator
- **Hodge theory**
  1. Hermitian differential geometry

2. The canonical connection and curvature of a Hermitian holomorphic vector bundle
3. Hermitian exterior algebra on a Hermitian vector space
4. Harmonic theory on compact manifolds
5. Representations of $sl(2, \mathbb{C})$ on Hermitian exterior algebras
6. Differential operators on a Kähler manifold
7. The Hodge decomposition theorem on compact Kähler manifolds
8. The Hodge-Riemann bilinear relations on a Kähler manifold

- **Characteristic classes**

1. Chern-Weil homomorphism
2. Invariant polynomials
3. Chern classes
4. Pontryagin classes
5. Euler's class