

Operads

Brief description: Operads, cooperads, the bar construction for operads and the cobar construction for cooperads. Koszul duality for (co)operads, homotopy algebras.

Prerequisites: Math 224, Math 227A, Math 246A

- Triples, algebras over triples, polynomial functors, operads, algebras over operads.
- Examples of linear operads: operad of commutative algebras, Lie algebras, and associative algebras.
- Cotriples, cooperads, coalgebras over cooperads. Examples.
- Free (co)algebras over linear (co)operads. Derivations.
- The (co)bar construction for linear (co)operads.
- Coalgebras over the bar construction. Dual description for the cobar construction.
- Twisting cochains. The adjunction between the categories of algebras and coalgebras.
- Koszul duality. Koszul operads.
- Ginzburg-Kapranov criterion of Koszulity.
- Examples of Koszul operads: Lie, Comm, Assoc.
- Homotopy algebras.

The course is based on the following texts:

References

- [1] B. Fresse, Koszul duality of operads and homology of partition posets, in "Homotopy theory and its applications (Evanston, 2002)", Contemp. Math. **346** (2004) 115–215.
- [2] E. Getzler and J.D.S. Jones, Operads, homotopy algebra and iterated integrals for double loop spaces, hep-th/9403055.
- [3] V. Ginzburg and M. Kapranov, Koszul duality for operads, Duke Math. J. **76**, 1 (1994) 203–272.
- [4] M. Markl, S. Shnider, and J. D. Stasheff, Operads in Algebra, Topology and Physics, AMS Bookstore, 2002

- [5] J.P. May, Operads, algebras and modules. Operads: Proceedings of Renaissance Conferences (Hartford, CT/Luminy, 1995), 15–31, Contemp. Math., 202, Amer. Math. Soc., Providence, RI, 1997.

Final project.

I will list 5-7 original research papers on operads and their applications. Each student will have to choose one of the papers read it and write a one-page review. This review should describe the results of the paper and what the student learnt from this paper. Several students may choose the same paper. In fact all students may choose the same paper. But they have to write the reviews independently.

Justification. This is a beautiful quickly developing subject of algebra. It has interesting applications in algebraic topology, Lie theory and geometry.