Workshop on Lie Groups, Lie Algebras and their Representations

January 17–18, 2009

University of California, Riverside

Titles and abstracts

Speaker : **Claudio Procesi**

Title : **From splines to the index theorem**

Abstract :

The celebrated index theorem of Atiyah–Singer led to a theory of transversally elliptic operators, with respect to some compact group of symmetries. In that case an explicit computation of the equivariant $K-$theory and the associated index for tori exists only in an implicit form. Some rather surprising connection arose with the work of Dahmen–Micchelli on partition functions and the box–spline which now allows us to make the constructions extremely explicit (work with De Concini, Vergne).

Speaker : **Eric Rains**

Title : **Vanishing integrals for Macdonald polynomials**

Abstract :

Every compact symmetric space G/H has an associated identity describing how to integrate an irreducible character of G over the subgroup H. (For instance $(U(n)/O(n))$, the integral of a Schur function over the orthogonal group is 0 unless all parts of the corresponding partition are even.) I'll discuss joint work with Monica Vazirani in which we generalize the $U(n)/O(n)$ and $U(2n)/Sp(2n)$ identities to the Macdonald polynomials, a well-studied two-parameter family of symmetric functions extending the Schur functions. The same approach also gives a number of other identities with a similar flavor, but which are of unknown representation-theoretic significance.

Speaker : **Tim Ridenour**

Title : **Ideals in parabolic subalgebras of simple Lie algebra**

Abstract :

We study $ad-nilpotent$ ideals of a parabolic subalgebra of a simple Lie algebra. Any such ideal corresponds to an antichain in a set of positive roots of the simple Lie algebra. We give a necessary and sufficient condition for an antichain to determine an $ad-nilpotent$ ideal of the parabolic. This result can be used to efficiently enumerate all

such ideals and allows us to recover a well-known result if D. Peterson when specializing to a Borel subalgebra.

Speaker : **Eric Sommers**

Title : **A duality map for nilpotent orbits**

Abstract :

For a simple Lie algebra $\mathfrak{g}$ and its Langlands dual $\mathfrak{g}'$, we define a certain set of nilpotent orbits in $\mathfrak{g} \times \mathfrak{g}'$. We discuss a duality map on this set, extending the duality map of Lusztig-Spaltenstein and explaining work of Achar.

Speaker : **Joshua Sussan**

Title : **The Affine Sergeev Algebra and the Lie Superalgebra of type Q**

Abstract :

The affine Sergeev algebra is a super version of the degenerate affine Hecke algebra. We construct finite dimensional integral modules for this algebra and give a complete combinatorial classification of the calibrated modules. A functor from category O of the Lie superalgebra of type Q to the category of integral representations of the affine Sergeev algebra is constructed. This is an extension of Sergeev duality between these two algebras.

Speaker : **Peter Tingley**

Title : **Three combinatorial models for affine $sl(n)$ crystals**

Abstract :

We present some combinatorial descriptions of affine $sl(n)$ crystals. These are similar to other models which appear in the literature, but the exact forms we use have some advantages; we see that the generating function of cylindric partitions is a specialization of the Weyl character formula, and also observe some well known forms of level-rank duality.

Speaker : **Rajeev Walia**

Title : **Tensor factorization and Spin construction for Kac-Moody algebras**

Abstract :

We will discuss the "Factorization Phenomenon" which occurs when a representation of a Lie algebra is restricted to a subalgebra, and the result factors into smaller representations of the subalgebra. We will analyze this phenomenon for symmetrizable Kac-Moody algebras.

We will provide an algebraic explanation for such a phenomenon using "Spin construction". We will present a few Factorization results for symmetrizable Kac-Moody algebra into another, and give some special cases for semi-simple finite dimensional Lie algebras. We will extend the notion of Spin functor, from finite-dimensional semi-simple Lie algebras to symmetrizable Kac-Moody algebras which requires a very delicate treatment. We will introduce a category of "d-finite, Orthogonal" $\mathfrak{g}$-representations for which, surprisingly, the Spin functor gives a $\mathfrak{g}$-representation in a category O_w, which is a weaker version Bernstein-Gelfand-Gelfand category O. If the input representation is additionally root finite, then Spin gives a representation in O. Also, for an integrable representation Spin produces an integrable representation. In O_w, the character determines the isomorphism class of a representation if it is integrable. We will give the formula for the character of Spin representation for the above category and work out the factorization results for an embedding of a finite dimensional semi-simple Lie algebra into its untwisted affine Lie algebra.

Speaker : **Nolan Wallach**

Title : **On the closure of the orbit of the centralizer of a regular element in a simple Lie algebra.**

Abstract :

It's vague but a more precise title would be too long. Obviously, the Grassmannian is $Gr_r(g)$ where r is the rank of g. This is joint work with Kostant.

Speaker : **Emilie Wiesner**

Title : **Ext-Groups for the Lie Algebra $W(1,1)$.**

Abstract :

In characteristic p, all restricted simple Lie algebras are classical or of Cartan type. Lie algebras of Cartan type fall into four classes of algebras, one of which contains the Witt algebras $W(1,1)$. Some progress has been made in describing the simple modules for $W(1,1)$. It particular, it is known that there are p restricted simple modules, that the modules all lie in one block, and what the dimensions of the modules are. A natural next question to ask is about extensions between the restricted simple modules. I will present some results addressing this question and talk about future directions.

Speaker : **Oded Yacobi**

Title : **An explicit Sp_n to $Sp_{n-1} \times Sp_1$ branching formula and a Gel'fand-Zeitlin basis of the symplectic group**

Abstract :

In the restriction of an irreducible representation of the rank n symplectic group Sp_n to the standard Sp_{n-1}, the multiplicity spaces are naturally SL_2-modules. We will describe a result (joint with N. Wallach) that identifies the multiplicity spaces as a specified tensor product of n irreducible SL_2-modules. We will then use this result to construct a Gel'fand-Zeitlin type basis for all irreducible representations of Sp_n.