Sample Problems for 10A Midterm

- The exam is on Friday, 11/04, 10:10 am – 11:00 am.
- In each problem, you have to show every step of your calculation.

1. Find the first and the second order partial derivatives:
   (1) \( f(x) = 3 \tan(2x) \).
   (2) \( f(x, y) = y + \cos^2 x \).
   (3) \( f(x, y, z) = x^2 + y^3 - e^{3z} \).

2. Are following functions continuous at \((0, 0)\)?
   (1) \( f(x, y) = xy^{10} + y - 2016x \).
   (2) \( f(x, y) = \frac{xy^{10} + yx^{10}}{x^2 + 2016y^2} \) and \( f(0, 0) = 0 \).
   (3) \( f(x, y) = \frac{xy}{x^2 + 2016y^2} \), \( f(0, 0) = \frac{1}{2017} \).

3. Find the derivative matrix \( Df(x, y) \):
   (1) \( f(x, y) = (x + 10xy, x) \).
   (2) \( f(u, v) = (u^2 + v, v - u^3), u = 2yx, v = y^2 \sin x + y^3 \).

4. Find the normal vector and the plane passing through the line \( x = 2t - 1, y = 2 + t, z = 2 \) and the point \((1, 0, 1)\).

5. Let \( \mathbf{u} = (1, 3, 2) \) and \( \mathbf{v} = (-1, 3, 5) \).
   (1) Find the dot and cross product of \( \mathbf{u} \) and \( \mathbf{v} \)
   (2) Find the length of \( \mathbf{u} \) and \( \mathbf{v} \);
   (3) Find the distance between \( \mathbf{u} \) and \( \mathbf{v} \);
   (4) Find the angle between \( \mathbf{u} \) and \( \mathbf{v} \).

6. Let \( f(x, y) = x^2 + y^2 \), find the graph and the level curves of this function.