Sample Problems for the Midterm

- The exam is on Wednesday, 02/16, 3:10 pm – 4:00 pm.
- In each problem, you have to show every step of your calculation.

1. Find the volumes of following regions:
   (1) under the surface \( z = f(x, y) = 3e^{2x+y} \) and above \( 0 \leq x, y \leq 2, z = 0 \).
   (2) under \( z = f(x, y) = y + \sin^{2}x \) and above \( z = 0, 1 \leq y \leq 2, 0 \leq x \leq \pi \).
   (3) inside \( x^2 + y^2 = 4 \) and \( 0 \leq x \leq z^2 \).

2. Find the following double integrals \( \iint_{D} f dA \):
   (1) \( f(x, y) = xy^2 + y - 2x, D = \{1 \leq x, y \leq 2\} \).
   (2) \( f(x, y) = xy^2 + yx^{100}, 0 \leq y \leq x \leq 1 \).
   (3) \( f(x, y) = xy, x^2 + 2y^2 \leq 2 \).

3. Find the triple integrals \( \iiint_{W} f(x, y, z) dV \):
   (1) \( f(x, y) = (x^2 + 2y)x, W = \{1 \leq x, y, z \leq 2\} \).
   (2) \( f(x, y, z) = y^2 \sin x + y^3 + z, 0 \leq x \leq y \leq z \leq 1 \).

4. Find the triple integrals \( \iiint_{W} f(x, y, z) dV \):
   (1) \( f(x, y, z) = e^{x^2+y^2+z}, x^2 + y^2 \leq z \leq \sqrt{x^2+y^2} \).
   (2) \( f(x, y, z) = \sin[(x^2 + y^2 + z^2)^{\frac{3}{2}}], 1 \leq x^2 + y^2 + z^2 \leq 4 \).
   (3) \( f(x, y, z) = \sin(x - y) + \cos(y + z), 1 \leq x \leq y \leq z \leq 2 \).