

Sample Problems for the Midterm

- The exam is on Tuesday, 05/03, 11:10 am – 12:30 noon.
- In each problem, you have to show every step of your calculation.

1. Find the volumes of following regions:

- (1) under the surface $z = f(x, y) = 3e^{2x+y}$ and above $0 \leq x \leq 1, 0 \leq y \leq 2, z = 0$.
- (2) under $z = f(x, y) = y + \sin^2 x$ and above $z = 0, 1 \leq y \leq 2, 0 \leq x \leq \pi$.
- (3) inside $x^2 + y^2 = 4$ and $x = z^2$.

2. Find the following double integrals $\iint_D f dA$:

- (1) $f(x, y) = xy^2 + y - 2x, D = \{0 \leq x \leq 1, 0 \leq y \leq 2\}$.
- (2) $f(x, y) = xy^2 + yx^{100}, 0 \leq y \leq x \leq 1$.
- (3) $f(x, y) = xy, x^2 + 2y^2 \leq 2$.

3. Find the triple integrals $\iiint_W f(x, y, z) dV$:

- (1) $f(x, y, z) = (x^2 + 2y)x, W = [1, 2] \times [1, 2] \times [1, 2]$.
- (2) $f(x, y, z) = y^2 \sin x + y^3 + z, 0 \leq x \leq y \leq z \leq 1$.

4. Find the triple integrals $\iiint_W f(x, y, z) dV$:

- (1) $f(x, y, z) = e^{x^2+y^2+z}, x^2 + y^2 \leq z \leq \sqrt{x^2 + y^2}$.
- (2) $f(x, y, z) = \sin[(x^2 + y^2 + z^2)^{\frac{3}{2}}], 1 \leq x^2 + y^2 + z^2 \leq 4$.
- (3) $f(x, y, z) = \sin(x - y) + \cos(y - z), 1 \leq x \leq y \leq z \leq 2$.

5. Assume that $\iint_{[0,1] \times [2,8]} f dA = 10, \iint_{[1,100] \times [2,8]} f dA = 2, \iint_{[0,1] \times [0,8]} f dA = -2$.

- (1) Find $\iint_{[0,100] \times [2,8]} 2f dA$;
- (2) Find $\iint_{[0,1] \times [0,2]} f dA$;
- (3) Is there a point in $[0, 1] \times [0, 2]$ such that $f \geq -6$?