Sample Problems for 10A Midterm

- *The exam is on Thursday, 02/17, 11:10 am – 12:30 pm.*
- *In each problem, you have to show every step of your calculation.*

1. Find the first and the second order partial derivatives:

    (1) $f(x) = 3\tan(2x)$.
    (2) $f(x, y) = y + \cos^2 x$.
    (3) $f(x, y, z) = x^2 + y^3 - e^{3z}$.

2. Are following functions continuous at $(0, 0)$?

    (1) $f(x, y) = xy^{10} + y - 2011x$.
    (2) $f(x, y) = \frac{xy^{10} + yx^{10}}{x^2 + 2011y^2}$ and $f(0, 0) = 0$.
    (3) $f(x, y) = \frac{xy}{x^2 + 2011y^2}$, $f(0, 0) = \frac{1}{2012}$.

3. Find the derivative matrix $Df(x, y)$:

    (1) $f(x, y) = (x + 10xy, x)$.
    (2) $f(u, v) = (u^2 + v, v - u^3)$, $u = 2yx$, $v = y^2 \sin x + y^3$.

4. Find the normal vector and the plane passing through the line $x = 2t - 1, y = 2 + t, z = 2$ and the point $(1, 0, 1)$.

5. Let $\mathbf{u} = (1, 3, 2)$ and $\mathbf{v} = (-1, 3, 5)$.

    (1) Find the dot and cross product of $\mathbf{u}$ and $\mathbf{v}$
    (2) Find the length of $\mathbf{u}$ and $\mathbf{v}$;
    (3) Find the distance between $\mathbf{u}$ and $\mathbf{v}$;
    (4) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

6. Let $f(x, y) = x^2 + y^2$, find the graph and the level curves of this function.