Math 10A Final Exam Winter 2006

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Your pledge: I pledge my honor that I have not violated the honor rule and done anything related to cheating during this examination.

Math 10A Final Exam

• This is a close book exam. The total points are 100+5.
• In each problem, you have to show every step of your calculation.

Name:____________________________
ID Number:________________________

1. (10 points) Find the first and the second order partial derivatives:
   
   (a) \( f(x, y) = e^{xy} + y^{10} \).
   
   (b) \( f(x, y, z) = x(z^2 - e^y)^{10} \).
2. (10 points) Are following functions continuous at \((0, 0)\)? Explain your reasons.

(a) \( f(x, y) = xy^{10} + y \sin^{10} 2x. \)

(b)

\[
f(x, y) = \begin{cases} 
\frac{x^2 + y^2}{2006x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\
\frac{2}{2007} & \text{if } (x, y) = (0, 0).
\end{cases}
\]

(c)

\[
f(x, y) = \begin{cases} 
\frac{x^{2006} + y^{10}}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0) \\
1 & \text{if } (x, y) = (0, 0).
\end{cases}
\]
3. (10 points) Find the derivative matrix $Df(x, y)$ for $f(u, v) = (u^2v, u + v^2)$, $u = \sin(x + y)$, $v = xy^2$. 
4. (10 points) Find a normal vector and the tangent plane of the surface $e^z + x - y \sin z = 1$ at point $(0, 1, 0)$. 
5. (10 points) Find the maximum of the function $f(x, y) = x^2 - y^2$ on the domain $9x^2 + y^2 \leq 10$. 
6. (10 points) Find the second order Taylor series of the function $f(x, y) = e^y + x \sin y$ at $(0, 0)$,
7. (10 points) Find all the critical points of \( f(x, y) = \cos(x + y) + y^3 \) and use the second derivatives to test the nature of them. Find the local maximal and minimal points.
8. (10 points) Assume the acceleration \( a(t) = (\sin t, 10t, t^2 - 1) \) and the initial position \( c(0) = (10, 0, 0) \). Find the position curve \( c(t) \) if the initial velocity is \( v(0) = (0, 0, 2006) \).
9. (10 points) Find the arc length of the curve \( c(t) = (13(t - \sin t), 12(1 - \cos t), 5(1 - \cos t)) \) with \( t \in [0, 2\pi] \).
10. (10+5 points) Find the divergence of the vector field \( F = (xyz, x^3, y^3) \).
Is \( F \) a curl of a vector field? (bonus problem) Is \( F \) a gradient of a function?