Math 138A First Exam

- *The exam is on Thursday, 02/15, 9:40 pm – 11:00 pm.*
- *In each problem, you have to show every step of your calculation.*

1. (20 points) Verify that $\alpha(t) = (t^2, t^3, t^4)$ is a regular curve for $t > 0$. Is it regular at $t = 0$?

2. (20 points) Is $X(r, \theta) = (ar\cos\theta, br\sin\theta, r + \theta)$ a regular surface for $r > 0$? Is it regular at $r = 0$?

3. (30 points) Let $\alpha(t) = (t^{\frac{3}{2}}, \sin t, \cos t)$

(1) Find the tangent vector and the normal vector.
(2) Find the curvature and torsion.
(3) Find the length of $\alpha([0, 2\pi])$.

4. (30 points) Find the first fundamental form of $X(u, v) = (u, v, u^2 + v^2)$. Find the area of S with $u^2 + v^2 \leq 1$.