

## Math 46, First Exam

Tuesday, 5/2, 11:10 am – 12:30 pm.

- *This is a close book exam. The total points are 100.*
- *In each problem, you have to show every step of your calculation.*
- *You are only allowed to use calculator to perform basic numerical calculations.*

**Name:** \_\_\_\_\_

**ID Number:** \_\_\_\_\_

1. (20 points) Verify that each given function is a solution of the given differential equation or initial value problem:

- (1) Function:  $y = y(x)$  satisfying the algebraic equation  $x^2 + y^2 = 1$ ; Differential equation:  $y' = -\frac{x}{y}$ .
- (2) Function:  $y = e^{-x} + e^{-2x}$ ; Initial value problem:  $y'' + 3y' + 2y = 0$ ,  $y(0) = 2$ ,  $y'(0) = -3$ .

2

2. (10 points) Solve the following differential equation:

$$y' \sin x = y^2 \cos x.$$

**3.** (15 points) Solve the following initial value problem:

$$(1 + e^x)y' + e^x y = 0, \quad y(0) = 1.$$

4. (20 points) An initial deposit of 1 million dollars in an investment account with 12% annual interest rate compounded continuously will approximately last how long if it is subject to annual withdrawals of 0.2 million dollars? (Here is a bonus question: What is the maximal amount of annual withdrawals you can make so that you could never empty your account?)

**5.** (20 points) A tank contains 50 gallon of pure water initially. A salt solution with a concentration of 0.1 lb/gal flows into the tank at a rate of 4 gal/min. The well-stirred mixture in the tank is then flows out at the rate of 5 gal/min. What will be the concentration of salt in the liquid flowing out of the tank when there are 30 gallons of liquid in the tank?

6

**6.** (15 points) Use the three-term Taylor series method:

$$y_{n+1} = y_n + hf(x_n, y_n) + \frac{h^2}{2}(f_x(x_n, y_n) + f_y(x_n, y_n)f(x_n, y_n))$$

to compute the approximation solution of the initial value problem

$$y' = xy, \quad y(0) = 1$$

at  $x_1 = 0.1$  and  $x_2 = 0.2$ .