

## Sample Problems for the Second Exam

- The exam is on Thursday, 11/30, 2:10 pm – 3:30 pm.
- In each problem, you have to show every step of your calculation.

1. Decide whether the following pairs of functions are linearly independent or not:

- (1)  $\sin(\pi + x)$  and  $\sin x$ ;
- (2)  $e^x$  and  $xe^x$ .

2. Solve the following second order linear equations:

- (1)  $2y'' - 3y' + y = 0$ ;
- (2)  $y'' - y = 8xe^x$ ;
- (3)  $y'' - 4y' + 3y = 20 \cos x$ .

3. Solve the following initial value problems:

- (1)  $y'' - 4y' + 4y = 0$ ,  $y(0) = 1, y'(0) = 1$ ;
- (2)  $y'' + 2y' + 2y = 0$ ,  $y(0) = 1, y'(0) = 0$ ;
- (3)  $y'' + y' - 2y = 2x$ ,  $y(0) = 0, y'(0) = 1$ .

4. Knowing that  $y = x$  and  $y = x^{-1}$  are two solutions of the differential equation

$$x^2y'' + xy' - y = 0,$$

- (1) find the solution of that equation satisfying  $y(1) = 0$  and  $y'(1) = 2$ ;
- (2) find the solutions of the nonhomogeneous equation

$$x^2y'' + xy' - y = x^{-2}.$$

5. Knowing that  $y = x$  is a solution of

$$f(x)y'' + xg(x)y' - g(x)y = 0.$$

- (1) Find all the solutions.
- (2) If  $g(x) = 1$  and  $f(x) = x^2$ , find all the solutions.

**6.** A 32 lb weight is attached to a frictionless spring, which in turn is suspended from the ceiling. The weight stretches the spring 2 ft and comes to a rest in its equilibrium position. The weight is then pulled down an additional 6 inches and released.

- (1) Find the resulting motion of the weight as a function of time.
- (2) Find the amplitude, phase angle, period and frequency of the resulting motion.
- (3) At what time does the weight first pass through the equilibrium position and what is the velocity at that time?
- (4) Sketch the graph of the motion of the weight.
- (5) Suppose the above spring-weight system is subject to a damping force (e.g., underwater) of 4 lb ft/sec, determine the motion of weight.
- (6) Is the damped system overdamped, underdamped, or critical damped?
- (7) sketch the graph of the motion of the weight.