Sample Problems for the Midterm

- The exam is on Friday, 02/19, 3:10 pm – 4:00 pm.
- In each problem, you have to show every step of your calculation.

Basic notions:
1. Check that each given function is a solution of the given problem or not:
   (1) Function: \( y = 3e^{2x} \); Differential equation: \( y' - 2y = 0 \).
   (2) Function: \( y = y(x) \) satisfying the algebraic equation \( x^2 + cy = 0 \), where \( c \) is a constant; Differential equation: \( y' = \frac{xy}{x^2-1} \).
   (3) Function: \( y = e^{-x} - e^{-2x} \); Initial value problem: \( y'' + 3y' + 2y = 0, y(0) = 0, y'(0) = 1 \).

First order equations:
2. Solve the following first order equations:
   (1) \( xy' + y - 2x = 0 \).
   (2) \( y' \sin x + y \cos x = 1 \).
   (3) \( xyy' = (y^2 - 1)^2 \).
   (4) \( (x^2 + y^2)dx + 2xydy = 0 \).
   (5) \( (3x + 2y^2)dx + 2xydy = 0 \).
   (6) \( y' = \frac{x^2 + 2xy + y^2}{x^2} \).
   (7) \( y' + y = xy^2 \).

3. Solve the following initial value problems:
   (1) \( y' + 2xy = x, \quad y(0) = 1 \).
   (2) \( 2yy' \sin x + y^2 \cos x = 1, \quad y(\pi/2) = 0 \).

Application problems:
4. An initial deposit of $1,000,000 in a bank with 6% annual interest rate compounded continuously will approximately last how long if it is subject to annual withdrawals of $100,000?

Approximation solutions:
5. Use Euler's method to compute the approximation solution of the initial value problem
   \[ y' = y + x, \quad y(0) = 1 \]
   at \( x_1 = 0.1 \) and \( x_2 = 0.2 \). Compare your approximation with the actual values.