Sample Problems for the Midterm

- *The exam is on Friday, 02/19, 3:10 pm – 4:00 pm.*
- *In each problem, you have to show every step of your calculation.*

Basic notions:

1. Check that each given function is a solution of the given problem or not:

(1) Function: $y = 3e^{2x}$; Differential equation: $y' - 2y = 0$.
(2) Function: $y = y(x)$ satisfying the algebraic equation $x^2 + cy = 0$, where c is a constant; Differential equation: $y' = \frac{xy}{x^2-1}$.
(3) Function: $y = e^{-x} - e^{-2x}$; Initial value problem: $y'' + 3y' + 2y = 0$, $y(0) = 0$, $y'(0) = 1$.

First order equations:

2. Solve the following first order equations:

(1) $xy' + y - 2x = 0$.
(2) $y' \sin x + y \cos x = 1$.
(3) $xyy' = (y^2 - 1)^2$.
(4) $(x^2 + y^2)dx + 2xydy = 0$.
(5) $(3x + 2y^2)dx + 2xydy = 0$.
(6) $y' = \frac{x^2+2xy+y^2}{x^2}$.
(7) $y' + y = xy^2$.

3. Solve the following initial value problems:

(1) $y' + 2xy = x, \qquad y(0) = 1.$
(2) $2yy' \sin x + y^2 \cos x = 1, \qquad y(\pi/2) = 0.$

Application problems:

4. An initial deposit of \$1,000,000 in a bank with 6% annual interest rate compounded continuously will approximately last how long if it is subject to annual withdrawals of \$100,000?

Approximation solutions:

5. Use Euler's method to compute the approximation solution of the initial value problem

$$y' = y + x, \qquad y(0) = 1$$

at $x_1 = 0.1$ and $x_2 = 0.2$. Compare your approximation with the actual values.