Sample Problems for the Midterm

• The exam is on Thursday, 02/16, 2:10 pm – 3:30 pm.

1. Find the solutions of the given problem:
   (1) \( y' - 2y = 0 \).
   (2) \( y' = \frac{xy}{x^2 - 1} \).
   (3) \( 3y' + 2xy^2 = 0, \quad y(0) = 1 \).
   (4) \( xyy' = (y^2 - 1)^2 \).

2. Solve the following first order linear equations:
   (1) \( y' - 2y = 0 \).
   (2) \( xy' + y - 2x = 0 \).
   (3) \( y' \sin x + y \cos x = 1 \).
   (4) \( y' + 2xy = x, \quad y(0) = 1 \).

3. Solve the following equations whenever it is exact:
   (1) \( (x^2 + y^2)dx + 2xydy = 0 \).
   (2) \( (3x + 2y^2)dx + 2xydy = 0 \).
   (3) \( (x^2 + 2xy)dx + x^2dy = 0 \).
   (4) \( ydx + xy^2dy = 0 \).

4. Solve the following Bernoulli equations:
   (1) \( y' + y = xy^2 \).
   (2) \( 2yy' \sin x + y^2 \cos x = 1, \quad y(\pi/2) = 0 \).
   (3) \( y' + y = y^2e^x \).
   (4) \( y' + \frac{2y}{x} = -x^9y^5, \quad y(-1) = 2 \).

5. Solve following homogeneous equations:
   (1) \( y' = \frac{x^2 + 2xy + y^2}{x^2} \).
   (2) \( y' = \frac{y-x}{x} \).
   (3) \( y' = \frac{x^2 + y^2}{2xy} \).

6. Use integrating factor method to solve following equations:
   (1) \( (3x + 2y^2)dx + 2xydy = 0, \quad y(0) = 1 \).
   (2) \( (y + 1)dx - xdy = 0 \).
   (3) \( ydx + 3xdy = 0 \).
   (4) \( 3x^2y^2dx + (2x^3y + x^3y^4)dy \).

7. Assume a radioactive material has 16 grams in 2000 and has 1 gram now, what is the half life of it? Give the amount of this material as a function of time \( t \) of years. What amount of the material will be still left 10 years later?