Sample Problems for the Midterm

- The exam is on Tuesday, 02/19, 2:10 pm – 3:30 pm.
- In each problem, you have to show every step of your calculation.

1. Find the solutions of the given problem:
   1. \( y' - 2y = 0. \)
   2. \( y' = \frac{xy}{x^2-1}. \)
   3. \( 3y' + 2xy^2 = 0, \quad y(0) = 1. \)
   4. \( xyy' = (y^2 - 1)^2. \)

2. Solve the following first order linear equations:
   1. \( y' - 2y = 0. \)
   2. \( xy' + y - 2x = 0. \)
   3. \( y' \sin x + y \cos x = 1. \)
   4. \( y' + 2xy = x, \quad y(0) = 1. \)
   5. \( y' = y + x, \quad y(0) = 1. \)

3. Solve the following equations whenever it is exact:
   1. \( (x^2 + y^2)dx + 2xydy = 0. \)
   2. \( (3x + 2y^2)dx + 2xydy = 0. \)
   3. \( (x^2 + 2xy)dx + x^2dy = 0. \)
   4. \( ydx + xy^2dy = 0. \)

4. Solve the following Bernoulli equations:
   1. \( y' + y = xy^2. \)
   2. \( 2yy' \sin x + y^2 \cos x = 1, \quad y(\pi/2) = 0. \)
   3. \( y' + y = y^2. \)
   4. \( y' + y = y^{-2}. \)
   5. \( y' + y = y^2e^x. \)
   6. \( y' + \frac{2y}{x} = -x^9y^5, \quad y(-1) = 2. \)

5. Solve following homogeneous equations:
   1. \( y' = \frac{x^2 + 2xy + y^2}{x^2}. \)
   2. \( y' = \frac{y-x}{x}. \)
   3. \( y' = \frac{x^2 + y^2}{2xy}. \)

6. Use integrating factor method to solve following equations:
   1. \( (3x + 2y^2)dx + 2xydy = 0, \quad y(0) = 1. \)
   2. \( ydx + xy^2dy = 0. \)
   3. \( (y + 1)dx - xdy = 0. \)
   4. \( ydx + 3xdy = 0. \)
   5. \( 3x^2y^2dx + (2x^3y + x^3y^4)dy. \)