Sample Problems for the Midterm

- *The exam is on Tuesday, 02/11, 2:10 pm – 3:30 pm.*

1. Find the solutions of the given problem:

(1) $y' - 2y = 0$.
(2) $y' = \frac{xy}{x^2-1}$.
(3) $3y' + 2xy^2 = 0, \qquad y(0) = 1$.
(4) $xyy' = (y^2 - 1)^2$.

2. Solve the following first order linear equations:

(1) $y' - 2y = 0$.
(2) $xy' + y - 2x = 0$.
(3) $y' \sin x + y \cos x = 1$.
(4) $y' + 2xy = x, \qquad y(0) = 1$.

3. Solve the following equations whenever it is exact:

(1) $(x^2 + y^2)dx + 2xydy = 0$.
(2) $(3x + 2y^2)dx + 2xydy = 0$.
(3) $(x^2 + 2xy)dx + x^2dy = 0$.
(4) $ydx + xy^2dy = 0$.

4. Solve the following Bernoulli equations:

(1) $y' + y = xy^2$.
(2) $2yy' \sin x + y^2 \cos x = 1, \qquad y(\pi/2) = 0$.
(3) $y' + y = y^2e^x$.
(4) $y' + \frac{2y}{x} = -x^9y^5, \qquad y(-1) = 2$.

5. Solve following homogeneous equations:

(1) $y' = \frac{x^2+2xy+y^2}{x^2}$.
(2) $y' = \frac{y-x}{x}$.
(3) $y' = \frac{x^2+y^2}{2xy}$.

6. Use integrating factor method to solve following equations:

(1) $(3x + 2y^2)dx + 2xydy = 0, \qquad y(0) = 1$.
(2) $(y + 1)dx - xdy = 0$.
(3) $ydx + 3xdy = 0$.
(4) $3x^2y^2dx + (2x^3y + x^3y^4)dy$.

7. Assume a radioactive material has 16 grams in 2000 and has 1 gram now, what is the half life of it? Give the amount of this material as a function of time t of years. What amount of the material will be still left 10 years later?