Sample Problems for the First Midterm

- *The exam is on Monday, 10/27, 1:10 pm – 2:00 pm.*
- *In each problem, you have to show every step of your calculation.*

Basic notions:

1. Find the limits:

(1) $\lim_{x\to\frac{\pi}{4}} 3\cos 2x$.
(2) $\lim_{x\to 0} x^2 + 1000$.
(3) $\lim_{x\to 1} \frac{x^3}{x^2+5}$.
(4) $\lim_{x\to 1} \frac{x^2+x-2}{x^3-1}$.
(5) $\lim_{x\to 1+} \frac{\sqrt{x-1}}{\sqrt{x+8}-3}$.
(6) $\lim_{x\to 0} \frac{3\cos 2x-\sin x-3}{x}$.

2. Are following functions continuous?

(1) $f(x) = x^2 + \sin 2x$.
(2) $y = 1$ if x is a rational number and $y = \cos x$ if x is irrational.

3. Find the derivatives:

(1) $\sin x + \tan x$.
(2) $3x(x^3 + 2003)^9$.
(3) $\sec(x^{-\frac{2}{3}} + 1)$.

Application problems:

4. Find the derivative of y if $x^2 + 3y^2x - y^3 = 3$. Find the tangent line at $(1, 1)$.

5. Find the velocity and the acceleration of a object with a position function $s = 3t^4 - 2t + 1$ at $t = 1$.