Sample Problems for the Second Midterm

- *The exam is on Friday, 11/21, 1:10 pm – 2:00 pm.*
- *In each problem, you have to show every step of your calculation.*

Basic notions:

1. Find the extrema of following fuctions:

(1) $3\cos 2x$ on $[1, 4\pi]$.
(2) $x^2 + 1000$ on $[-1, 1]$.
(3) $x^3 + x^2 + 5$ on $[-4, 6]$.
(4) $(x^2 + x - 2)^3$ on $[2, 5]$.

2. Find the critical points and inflection points of following functions: On which part of interval the functions are increasing? On which part of interval the functions are concave upward? Find the intercepts and asymptotics

(1) $f(x) = x^2 + 2x$.
(2) $y = \frac{x^2}{x+1000}$.

3. Apply the mean value theorem to following functions:

(1) $3x(x^2 + 2)^2$ on $[-1, 1]$.
(2) $x^3 + 1$ on $[1, 3]$.

4. Find the limits:

(1) $\lim_{x\to\infty} \frac{x^2+1}{x-x^2}$.
(2) $\lim_{x\to\infty} \sin x$.
(3) $\lim_{x\to\infty} \frac{\sin x}{x}$.

5. Find the differential dy with $y = x^{\frac{3}{2}} - 7$. Estimate $(3.5)^{\frac{3}{2}} - 7$. Apply Newton's method to estimate a zero between 1 and 4.

Application problems:

6. Find the dimension of the smallest area for a page with 20 square inches print and 1 inch margins on the top and the bottom, 0.8 inches margins on the right and left.