Errata for 'The degree of the divisor of jumping rational curves' by Z. Ran (Quart. J. Math. 52 (2001), 367-383)

There is an inaccuracy in §3, in that the 'twisting divisor' D is not chosen carefully enough. Correcting for this, we end up with an improved, substantially simpler formula in the main result, Theorem 3.1.

We use the notations of the paper, especially §3. In the setup described there, it is possible for the Fitting scheme $\mathrm{Fit}_0(R^1\pi_*(G))$ to be nonreduced at critical values of π, i.e. points $b \in B$ such that $\pi^{-1}(b)$ is reducible. In this case the correction terms given in the paper don't correct as they should. However, this problem is easily remedied by broadening the choice of twisting divisor D to include linear combinations of special sections s_i *and* fibre components (rather than just s_i's).

Claim. *There is an integral linear combination D of sections s_i and fibre components of X/B such that for $G = f^*(E)(-D)$, we have*

$$H^1(G|_{F_b}) = 0$$

for every reducible fibre F_b.

If the claim is proven then, with such choice of D, no correction terms for the reducible fibres are necessary, so Theorem 3.1 holds with $h(t., a.) = 0$.

Proof of claim. The idea is to choose D of total fibre degree $\frac{ad}{r}+1$ with the property that, for each reducible fibre $F_b = X_1 \cup X_2$, some choice of ordering of the components X_1, X_2, and $d_i = \deg(f(X_i)), i = 1, 2$, we have that

$$D.X_1 = \lceil \frac{ad_1}{r} \rceil,$$

in which case automatically

$$D.X_2 = \lfloor \frac{ad_2}{r} \rfloor + 1.$$

It is clear that the latter conditions can be achieved by suitably adjusting a given D of the right fibre degree with fibre components. A specific choice of D is

$$D = (\frac{ad}{r}+1)s_1 - \sum_{F\in\mathcal{F}_1} \lceil \frac{aL.F}{r} \rceil F,$$

where $\mathcal{F}_1$ is the set of fibre components not meeting s_1.

Choosing such D and setting $G = f^*(E)(-D)$, almost balancedness of $G|_{X_i}$ implies that its splitting type is of the form $(0^{m_i}, (-1)^{r-m_i})$ for some $m_i, i = 1, 2$. Then the almost balancedness of $G|_{X_1\cup X_2}$ implies that it is either a sum of line bundles of bidegrees $(0,0), (0,-1), (-1,0)$, or a sum of line bundles of bidegrees $(0,-1), (-1,0), (-1,-1)$; but the second alternative would contradict the fact that the fibre degree of D is $\frac{ad}{r}+1$. Hence the first alternative holds, so that $H^1(G|_{X_1\cup X_2}) = 0$. $\square$

The formula for $L.s_i$ on p.380 l.3 should read

$$L.s_i = N_d(a_1, ..., a_i + 1, ...).$$

This is zero when $n = 2$ but not in general.

The formula for $L.R_i, n = 2$, on p.380, l.-12 should read

$$L.R_i = \sum_{d_1+d_2=d} \binom{3d-2}{3d_1-1} d_1 d_2^2 N_{d_1} N_{d_2}, \ n = 2$$

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