1. Suppose $S \subseteq \mathbb{R}$ and for some $c > 0$ we have

$$|s - t| \leq c$$

for all $s, t \in S$. Prove that

$$\sup S - \inf S \leq c.$$

(By the way, the converse is also true.)

2. Suppose that $a < c < b$ and $f \in \mathcal{R}[a, b]$. Prove that $f \in \mathcal{R}[a, c]$, $f \in \mathcal{R}[c, b]$ and

$$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx.$$

3. Suppose that $f, g \in \mathcal{R}[a, b]$ and $f(x) \leq g(x)$ for all $x \in [a, b]$. Show that

$$\int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx.$$

(Problems 2 and 3 are parts of Theorem 6.12 that Rudin does not prove. He does, however, give a hint for one of these problems.)