1. **Homomorphisms as functors.**

   A functor $F : G \to H$ is such that $F(g : x \to y) : F(x) \to F(y)$, compatible with composition

   $$F(g_1 : x \to y) \circ F(g_2 : y \to z) = F(g_1 \circ g_2 : x \to z) : F(x) \to F(z)$$

   If $G, H$ are groupoids with one object,

   $$F(g_1) \circ F(g_2) = F(g_1 \circ g_2)$$

   which means $F$ is a homomorphism from the morphism group of $G$ to that of $H$. Conversely, any group homomorphism defines a functor.

2. **Conjugation as a natural transformation.**

   A natural transformation $\alpha$ between functors $F_1, F_2 : G \to H$ assigns an endomorphism $\alpha_x : F_1(x) \to F_2(x)$ to each object $x$ of $G$ in such a way that that, for every $g : x \to y$ in $G$, $\alpha_x F_2(g) = F_1(g) \alpha_y$ in $H$.

   If $G$ and $H$ are groupoids with a single object, a natural transformation between group homomorphisms $F$ and $G$ is a single group element $\alpha \in H$ such that

   $$\alpha F_2(g) = F_1(g) \alpha \quad \text{for all} \quad g \in G,$$

   that is, $F_1(g)$ is the result of conjugating $F_2(g)$ by $\alpha \in H$.

3. **The center as the natural automorphisms of the unit.**

   If, now, $F_1 = F_2 = 1_G : G \to G$, a natural transformation $\alpha : F_1 \Rightarrow F_2$ is a group element $\alpha \in G$ satisfying

   $$\alpha g = g \alpha \quad \text{for all} \quad g \in G,$$

   that is, $\alpha$ is in the center of the group $G$.

4. **Group representations as functors.**

   If $G$ is a groupoid with one object, a functor $F : F \to \text{Vect}$ is a choice of a vector space $V = F(\bullet)$ and a group homomorphism $F : G \to \text{End}(V)$. In other words, $F$ is a representation of the group $G$.

5. **Intertwiners as natural transformations.**

   Suppose that $F, F'$ are two representations of the group $G$. Then, a natural transformation $\alpha : F \Rightarrow F'$ is a linear map $\alpha : V \to V'$ such that

   $$\alpha F'(g) = F(g) \alpha \quad \text{for all} \quad g \in G.$$

   The map $\alpha$ is called an intertwining operator between the representations $F$ and $F'$.