Any $H : T^*M \to \mathbb{R}$ gives the Hamiltonian vector field

$$v_H = \frac{\partial H}{\partial p_i} \frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i} \frac{\partial}{\partial p_i}$$

We have a 2-form $\omega$ on $T^*M$:

$$\omega = dq^i \wedge dp_i$$

Show

$$dH = \omega(v_H, -)$$

\[
\begin{align*}
\omega(v_H, -) &= dq^i(v_H)dp_i - dp_i(v_H)dq^i \\
&= dq^i \left( \frac{\partial H}{\partial p_i} \frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i} \frac{\partial}{\partial p_i} \right) dp_i - dp_i \left( \frac{\partial H}{\partial p_i} \frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i} \frac{\partial}{\partial p_i} \right) dq^i \\
&= \left( \frac{\partial H}{\partial p_i} dq^i \frac{\partial}{\partial q^i} \frac{\partial}{\partial p_i} - \frac{\partial H}{\partial q^i} dq^i \frac{\partial}{\partial p_i} \right) dp_i - \left( \frac{\partial H}{\partial p_i} dp_i \frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i} dp_i \frac{\partial}{\partial p_i} \right) dq^i \\
&= \frac{\partial H}{\partial p_i} dp_i + \frac{\partial H}{\partial q^i} dq^i \\
&= dH
\end{align*}
\]