1. Construct a direction field of the equation $y' = 2t - 4y$. Sketch a graph of the integral curve satisfying $y(0) = 1$.

2. What is the order of the following differential equation and why?
$$t^2 y'''' + y' \cos y'' = 3t \sin y^4.$$  

3. Find the general solution of the equation $y''' - 5y' + 6y = 0$.

4. Answer the following two questions for the equation $y' = y^2 - 5y + 6$.
   a) Which solutions are called equilibrium solutions?
   b) Find the equilibrium solutions.

5. a) Why doesn’t the uniqueness Theorem imply that the equation $ty' - y = t^2 \cos t$, $y(0) = 0$, has a unique solution?
   b) Show that every solution of $ty' - y = t^2 \sin t$, $y(\pi) = C$ is a solution on $(-\infty, +\infty)$ and satisfies $y(0) = 0$.

6. Solve the homogeneous equation $y' = \frac{x+y}{x-y}$.

7. Suppose that a substance decays at a yearly rate equal to half the square of the mass of the substance present. If we start with 50 g of the substance, how long will it be until only 25 g remain?

8. A space probe is to be launched from a space station 180 miles above Earth. Determine its escape velocity. Take Earth’s radius to be 3960 miles.

9. Find all solutions on the interval $(0, 2)$ of the initial problem $ty' + 2y = 0$, $y(1) = 1$

10. Is it correct to claim that by the uniqueness theorem for an ODE the only solution on the interval $(-\infty, +\infty)$ of the equation
$$y' = \sqrt{|y|}, \quad y(1) = 0$$
is the function $y(t) \equiv 0$?

11. Find all solutions on the interval $(0, 2)$ of the initial problems $t^3y' + 2(t+y)y^2 = 0$, $y(1) = 0$.

12. Solve the initial problem
$$(t^2 - 1)x' + 2tx = 0, \quad x(2) = \frac{1}{3}.$$Is the equation exact? Is it separable?

13. Solve the Bernoulli equation $y' - y = ty^{1/3}$, $y(0) = -8$.

14. Determine whether the functions $y_1 = e^{2t}$ and $y_2 = e^{-2t}$ form a fundamental set for the equation $y'' - 4y = 0$. 

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15. Is there a second order linear equation with constant coefficients for which \( y_1 = t \) and \( y_2 = \sin t \) are a fundamental set of solutions. Hint: Use Abel’s Theorem.

16. Find the solution of \( y'' + 5y' - 6y = 0 \), \( y(1) = 0 \), \( y'(1) = 1 \).

17. Find the solution of \( y'' - 6y' + 9y = 0 \), \( y(0) = 2 \), \( y'(1) = e^3 \).

18. Find the solution of \( y'' - y' + 2y = 0 \), \( y(0) = 1 \), \( y'(0) = 1 \).

19. A 2-lb weight stretches a spring 6 inches in equilibrium. An external force \( F(t) = \sin 8t \) is applied to the weight, which is released from rest 2 inches below equilibrium. Find its displacement for \( t > 0 \).

20. Two objects suspended from identical springs are set into motion. The period of one object is twice the period of the other. How are the weights of the two objects related?

21. A 6-lb weight stretches a spring 6 inches in equilibrium. Suppose that an external force \( F(t) = (3/16) \sin \omega t + (3/8) \cos \omega t \) lb is applied to the weight. For what value of \( \omega \) will resonance occur?

22. A 96-lb weight stretches a spring 3.2 ft in equilibrium. It is attached to a dashpot with damping constant \( c = 18 \) lb-s/ft. The weight is initially displaced 15 inches below equilibrium and given a downward velocity of 12 ft/s. Find its displacement for \( t > 0 \).

23. Find the general solution of \( y'' + 4y = (2t + 1) \sin 2t \).

24. Given that \( y_1 = t \) and \( y_2 = t \log t \) are two solutions of the equation
   \[ t^2 y'' - ty' + y = 0, \quad t > 0, \]
   find the general solution of the equation
   \[ t^2 y'' - ty' + y = 1, \quad t > 0, \]
   using the variation of parameters method. Recall, one of the steps is to find \( y_p(t) = u_1(t)y_1 + u_2(t)y_2 \), s.t., \( y_1u_1' + y_2u_2' = 0 \), which leads to a system for \( (u_1, u_2) \) involving the Wronskian matrix of \( y_1, y_2 \). Hint: Don’t forget to check that \( y_1 \) and \( y_2 \) are linearly independent.

25. Determine the Laplace transform \( \tilde{y} \) (not the solution \( y \) !) of the solution of the equation
   \[ y'' + y' + y = f(t), \quad y(0) = y'(0) - 1 \]
   where \( f(t) = \begin{cases} e^{-t}, & 0 \leq t < 1 \\ e^{-2t}, & t \geq 1 \end{cases} \).

26. Find the inverse Laplace transform of the function
   \[ F(s) = \frac{4}{(s^2 - 1)(s^2 + 1)}. \]

27. Use the Laplace transform to solve the initial value problem.
   \[ y'' + y = \sin 2t, \quad y(0) = y'(0) = 1. \]