1. Define a function \( \log : (0, \infty) \to \mathbb{R} \) by

\[
\log(x) = \int_{1}^{x} \frac{dt}{t}
\]

then
i) \( \text{dom } \log = \) 
ii) \( \text{range } \log = \)
iii) \( \text{im } \log = \)
iv) \( \text{log is injective } (T/F)? \)
v) \( \text{log is surjective } (T/F)? \)
vi) \( \text{log is bijective } (T/F)? \)
vii) \( \log([1,2]) = \)
vii) \( \log^{-1}([-1,1]) = \) (Give carefully, with domain and range specified.)

2. The function \( f : \mathbb{R} \to [-1,1] \) defined by \( f(x) = \sin x \) is not injective hence does not have an inverse. But there is such a function as \( \sin^{-1} \), also known as arcsin \( x \). How can this be? By cutting down the domain we are able to obtain a restriction of our function \( f \) which is injective but with same image. It is now bijective and so has an inverse which is, by definition, \( \sin^{-1} \). There are many ways to do this. How was it done in the Calculus? What are the domain and range of \( \sin^{-1} \)?

3. Prove that if \( m \) and \( n \) are integers then so is their sum \( m + n \). (Algebraists say that the integers are “closed under addition.”)

4. Prove that given any integer \( n \in \mathbb{Z} \) there is no integer \( m \) satisfying \( n < m < n + 1 \).

This is not assigned. It is another problem going around. It is easier than the first one I gave. Solve it and get your name in print! I will help. Let \( I_n = \{1,2,\ldots,n\} \). If \( A \) is a non-empty subset of \( I_n \), let \( n(A) \) denote the number of elements of \( A \) and let \( \Pi(A) \) denote the product of the integers in \( A \). Find, as a function of \( n \), the values of the sums

\[
\sum_1 \frac{1}{\Pi(A)} \text{ and } \sum_1 \frac{(-1)^{n(A)}}{\Pi(A)}
\]

where the summations are over the collection of all non-empty subset of \( I_n \).

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