1. The velocity of a body moving along the $x$-axis is $v = 2t^2 - 7t + 6$. When is the body moving forward?

We need to determine when $v > 0$. This will tell us when the body is moving forward. Note, $v = 2t^2 - 7t + 6$ factors as $(2t - 3)(t - 2)$. Hence, the critical values of $s$ are $t = \frac{3}{2}$ and $t = 2$. If we test $v$ on $[0, \frac{3}{2})$ we see $v > 0$. Testing $v$ on $(\frac{3}{2}, 2)$ we see that $v < 0$. Testing $v$ on $(2, \infty)$ we see that $v > 0$. Hence, the body is moving forward when $t < \frac{3}{2}$ and when $t > 2$.

2. Suppose for functions $f$ and $g$, we know $f(1) = 5, g(1) = -1.5, f'(1) = 11, g'(1) = -8, f(2) = 3, g(2) = -1, f'(2) = .3, g'(2) = -5$. Find the derivative of

(a) $f(x + g(x))$ at $x = 2$

(b) $g(f(x) + 2g(x))$ at $x = 1$.

(a) If $h(x) = f(x + g(x))$, then $h'(x) = f'(x + g(x))(1 + g'(x))$. Hence,

$$h'(2) = f'(2 + g(2))(1 + g'(2)) = f'(1)(1 - 5) = 11(-4) = -44.$$ 

(b) If $h(x) = g(f(x) + 2g(x))$, then $h'(x) = g'(f(x) + 2g(x))(f'(x) + 2g'(x))$. Hence,

$$h'(1) = g'(f(1) + 2g(1))(f'(1) + 2g'(1)) = g'(5 + 2(-1.5))(11 + 2(-8)) = -5(-5) = 25.$$ 

3. Find the tangent line to the parametric curve defined by $x = \tan(2t), y = \sec^2(t)$ when $t = -\frac{\pi}{6}$.

First we need to find $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$. We have $\frac{dy}{dt} = 2 \sec t \tan t \sec t = 2 \sec^2 t$ and $\frac{dx}{dt} = 2 \sec^2(2t)$. So $\frac{dy}{dx} = \frac{\sec^2 t \tan t}{2 \sec^2(2t)}$. The slope of the tangent line at $t = -\frac{\pi}{6}$ is $\frac{dy}{dx}(-\frac{\pi}{6}) = \frac{\frac{4}{3} - \frac{1}{\sqrt{3}}}{4} = -\frac{1}{3\sqrt{3}}$.

Therefore, the tangent line is $y - \frac{4}{3} = -\frac{1}{3\sqrt{3}}(x + \sqrt{3})$ or $y = -\frac{1}{3\sqrt{3}}x + 1$. 

\[1\]
4. If \( x^2 + y^3 = 5 \), find \( \frac{d^2x}{dy^2} \) at the point \((-2, 1)\).

First we need to find \( \frac{dx}{dy} \). Note, this is taking the derivative of everything with respect to \( y \).
\[
2x \frac{dx}{dy} + 3y^2 = 0. \text{ Hence, } \frac{dx}{dy} = -\frac{3y^2}{2x}.
\]

Now we take the derivative of \( \frac{dx}{dy} \) with respect to \( y \) again to get
\[
\frac{d^2x}{dy^2} = \frac{-12xy + 6y^2 \frac{dx}{dy}}{4x^2} = -\frac{12xy}{4x^2} - \frac{9y^4}{x}. \]

Now plug in the point \((-2, 1)\) to get
\[
\frac{d^2x}{dy^2}(-2, 1) = \frac{57}{32}.
\]

5. Two planes are flying at 35,000 feet along straight line courses that intersect at right angles. Plane A is approaching the intersection at a speed of 300 miles per hour and plane B is approaching the intersection at a speed of 400 miles per hour. At what rate is the distance between the planes changing when A is 50 miles from the intersection point and B is 120 miles from the intersection point?

Draw a right triangle ABC, where the right angle is at vertex C. Label side AC as \( x \), BC as \( y \) and AB as \( z \).

Note we have the relation \( x^2 + y^2 = z^2 \).

As the planes fly toward the intersection point C, all of the sides of the triangle are decreasing.

Take the derivative of the expression above with respect to \( t \) to get
\[
2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}.
\]

We know that \( \frac{dx}{dt} = -300 \text{ mph, } \frac{dy}{dt} = -400 \text{ mph, } x = 50 \text{ and } y = 120 \). 50, 120, 130 is a pythagorean triple so \( z = 130 \).

Plugging in all the known quantities we see that
\[
\frac{dz}{dt} = \frac{-15,000 - 48,000}{130} = -485 \text{ mph.}
\]

Hence, the distance between the planes is decreasing at a rate of 485 mph.

6. The edge of a cube is measured as 2 inches with an error of 1%. The cube’s volume is to be computed from this measurement. Estimate the percentage error in the volume computation.

The volume of a cube is given by \( V = s^3 \). The percentage error is given by \( \frac{dV}{V} \cdot 100\% \). \( dV = 3s^2 ds \) and \( \frac{dV}{V} = 3 \frac{ds}{s} \). But \( \frac{ds}{s} = .01 \). Hence, \( \frac{dV}{V} \cdot 100\% = 3\% \)

7. Find the absolute extrema of \( f(x) = 1 - 2\sqrt[3]{x^2} \) on the interval \([-8, 1]\).

The extrema can occur at the endpoints or at the critical values. The critical values are found by taking the derivative and setting it to 0 or finding where it is undefined.

\( f'(x) = \frac{-4x}{3\sqrt[3]{x^2}} \). \( f' \) is never 0 but 0 is a critical value as \( f'(0) \) is undefined. Now compare \( f(-8) = -7, f(0) = 1 \) and \( f(1) = -1 \). We see that the maximum is 1 and the minimum is -7.
8. Use Rolle’s Theorem to show that \( f(x) = \sqrt{x(4-x)} \) has a horizontal tangent line on the interval \([0, 4]\). Then find the \( x \) value where \( f \) has a horizontal tangent line.

Note that the domain of \( f \) is \([0, 4]\) and \( f \) is continuous on the whole domain and differentiable on \((0, 4)\), as \( f'(x) = \frac{2-x}{\sqrt{x(4-x)}} \), which is not defined at 0 or 4. \( f(0) = f(4) = 0 \). As we have all of the hypotheses of Rolle’s Theorem, we can conclude that \( f'(c) = 0 \) for some \( c \in (0, 4) \). Observing the derivative above, we see that \( c = 2 \).

9. Find the intervals where \( f(x) = x^3 - 2x^2 + 4 \) is increasing.

\( f'(x) = 3x^2 - 4x = x(3x - 4) \). Hence the critical values of \( f \) are \( x = 0 \) and \( x = \frac{4}{3} \). We break the number line into the following three intervals to test where the derivative is positive or negative.

\[
\begin{array}{ccc}
(-\infty, 0) & (0, \frac{4}{3}) & (\frac{4}{3}, \infty) \\
\begin{array}{c}
f'(-1) = 7 \\
f'(0) = -1 \\
f'(2) = 4 \\
f'(3) = 176
\end{array}
\end{array}
\]

Looking at the table we see that \( f \) is increasing on the intervals \((-\infty, 0] \) and \([\frac{4}{3}, \infty) \).

10. Find the \( x \) values where \( f(x) = (x+1)^3(x-2)^2 \) has critical points. Using the first derivative test, identify the critical values as either having a local maximum, a local minimum or neither.

\( f'(x) = 2(x+1)^3(x-2)+3(x+1)^2(x-2)^2 = (x+1)^2(x-2)[2(x+1)+3(x-2)] = (x+1)^2(x-2)(5x-4) \).

We see the critical values of \( f \) are \( x = -1, x = \frac{4}{5}, x = 2 \). We break the number line into the following four intervals to test where the derivative is positive or negative.

\[
\begin{array}{cccc}
(-\infty, -1) & (-1, \frac{4}{5}) & (\frac{4}{5}, 2) & (2, \infty) \\
\begin{array}{c}
f'(-2) = 56 \\
f'(0) = 8 \\
f'(1) = -4 \\
f'(3) = 176
\end{array}
\end{array}
\]

As there is no sign change at \( x = -1 \), \( f \) has neither a local maximum nor a local minimum at \( x = -1 \). As the sign changes from positive to negative at \( x = \frac{4}{5} \), \( f \) has a local maximum at \( x = \frac{4}{5} \). As the sign changes from negative to positive at \( x = 2 \), \( f \) has a local minimum at \( x = 2 \).

11. Find the intervals where \( f(x) = 6x^2 - x^4 \) is concave down.

Concavity is determined by the second derivative. \( f'(x) = 12x - 4x^3 \) and \( f''(x) = 12 - 12x^2 \). The \( x \) values where the concavity may change are \( x = -1, 1 \). We break the number line into the following three intervals to test where the second derivative is positive or negative.

\[
\begin{array}{ccc}
(-\infty, -1) & (-1, 1) & (1, \infty) \\
\begin{array}{c}
f''(-2) = -36 \\
f''(0) = 12 \\
f''(2) = -36
\end{array}
\end{array}
\]

Hence, \( f \) is concave down on the intervals \((-\infty, -1] \) and \([1, \infty) \).