1. For each matrix $A$ and ordered basis $\beta$, find $[L_A]_\beta$. Here $L_A$ denotes the linear operator on $\mathbb{R}^n$ defined by $A$ and $[L_A]_\beta$ denotes the matrix representation of the linear operator $L_A$ in the ordered basis $\beta$.

(a) 

$$A = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}, \beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$$

(b) 

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$

2. For each matrix $A \in \text{Mat}_{n\times n}(\mathbb{C})$ given below, determine the eigenvalues and the set of eigenvectors corresponding to each eigenvalue. Then in each case, find a basis for $\mathbb{C}^2$ consisting of eigenvectors. Finally, determine an invertible matrix $Q$ and a diagonal matrix $D$ such that $Q^{-1}AQ = D$.

(a) 

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$$

(b) 

$$B = \begin{pmatrix} i & 1 \\ 2 & -i \end{pmatrix}.$$

3. Let $T$ be a linear operator on a vector space $V$.

(a) Suppose $T$ is invertible and prove that a scalar $\lambda$ is an eigenvalue of $T$ if and only if $\lambda^{-1}$ is an eigenvalue of $T^{-1}$.

(b) Suppose that $x$ is an eigenvector of $T$ corresponding to the eigenvalue $\lambda$. For any positive integer $m$, prove that $x$ is an eigenvector of $T^m$ corresponding to the eigenvalue $\lambda^m$. 

1