1. (a) What is the definition of a diagonalizable linear operator $T$ on a finite dimensional vector space $V$?

(b) What is the definition of a diagonalizable $n \times n$-matrix?

(c) For each of the following matrices $A \in \text{Mat}_{n \times n}(\mathbb{R})$, determine if $A$ is diagonalizable (over $\mathbb{R}$), and if it is diagonalizable, find a matrix $Q$ such that $Q^{-1}AQ$ is a diagonal matrix.

\[
(a) \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad (b) \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}, \quad (c) \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}
\]

\[
(d) \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}, \quad (e) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}
\]

2. True/False: Determine which of the following statements are true. For each true statement, give a proof and for each false statement produce a counterexample.

(a) Any linear operator on an $n$-dimensional vector space that has fewer than $n$ distinct eigenvalues is not diagonalizable.

(b) Eigenvectors corresponding to the same eigenvalue are always linearly dependent.

(c) If $\lambda$ is an eigenvalue of a linear operator $T$, then each element of the eigenspace $E_\lambda$ is an eigenvector of $T$.

(d) If $\lambda_1$ and $\lambda_2$ are distinct eigenvalues of a linear operator $T$, then $E_{\lambda_1} \cap E_{\lambda_2} = \{0\}$.

(e) Let $A \in M_{n \times n}(\mathbb{F})$ and $\beta = \{v_1, \ldots, v_n\}$ be an ordered basis for $\mathbb{F}^n$ consisting of eigenvectors of $A$. If $Q$ is the $n \times n$ matrix whose $i$-th column is $v_i$ ($1 \leq i \leq n$), then $Q^{-1}AQ$ is a diagonal matrix.