1. Let $T$ be a diagonalizable linear operator on a finite-dimensional vector space $V$, and let $W$ be a $T$-invariant subspace of $V$. Suppose that $v_1, v_2, \ldots, v_k$ are eigenvectors of $T$ corresponding to distinct eigenvalues. Prove that if $v_1 + v_2 + \ldots + v_k$ is in $W$, then $v_i \in W$ for all $i$. (Hint: Use induction on $k$ and the trick we used to show that eigenvectors with distinct eigenvalues are linearly independent.)

2. Prove that the restriction of a diagonalizable linear operator $T$ to any nontrivial $T$-invariant subspace is also diagonalizable. (Hint: Use the result from the previous problem.)

3. Prove that if $T$ and $U$ are diagonalizable linear operators on a finite-dimensional vector space $V$ such that $UT = TU$, then $T$ and $U$ are simultaneously diagonalizable. (See problem 3 of homework 5 for the definition of simultaneously diagonalizable.) Hint: For any eigenvalue $\lambda$ of $T$ show that the eigenspace $E_\lambda$ is $U$-invariant, and then apply the previous problem to obtain a basis for $E_\lambda$ made up of eigenvectors for $U$. 