Section 3.3, Exercise 2 Solution

Let \( s(t) = 6t - t^2 \) be the position of a body moving on a coordinate line over the interval \( 0 \leq t \leq 6 \), with \( s \) in meters and \( t \) in seconds.

(a): Find the body’s displacement and average velocity for the time interval.

Displacement = \( s(6) - s(0) = (6(6) - 6^2) - (6(0) - 0^2) = 0 - 0 = 0 \).

Average velocity = \( \frac{\text{displacement}}{\text{change in time}} = \frac{0}{6-0} = 0 \).

(b): Find the body’s speed and acceleration at the endpoints of the interval.

To find speed we must first find velocity \( v(t) \), since speed = \( |v(t)| \). Velocity is given by the first derivative: \( v(t) = s'(t) = 6 - 2t \). Acceleration \( a(t) \) is given by the second derivative: \( a(t) = -2 \). At the left endpoint, \( t = 0 \), we have

\[
|v(0)| = 6 \text{ m/s}, \quad a(0) = -2 \text{ m/s}^2.
\]

At the right endpoint \( t = 6 \), we have

\[
|v(6)| = 6 \text{ m/s}, \quad a(6) = -2 \text{ m/s}^2.
\]

(c): When, if ever, during the interval does the body change direction?

The body will change direction when the velocity changes from a positive to a negative value; this happens precisely when the velocity = 0. We can find this time by setting \( v(t) = 0 \) and solving for \( t \):

\[
0 = v(t) = 6 - 2t
\]

so body changes direction at \( t = 3 \) seconds.