Plane sections of a surface

It is sometimes useful to analyze surfaces by means of *plane sections*, which are given by intersecting the surface with a plane which contains a normal line to a point \( p \) on the surface. The drawing below suggests that such an intersection is a curve, and in fact this is always true, at least near the point where the surface and line meet each other.

![Diagram of plane section](http://www.learner.org/courses/mathilluminated/images/units/8/1832.png)

(Source: [http://www.learner.org/courses/mathilluminated/images/units/8/1832.png](http://www.learner.org/courses/mathilluminated/images/units/8/1832.png))

We can use the results of Section III.6 to prove the intersection is locally a curve because the surface and the plane intersect transversely. The latter is true because the plane contains the normal line to the surface at the point \( p \), which means that the normal to the plane is perpendicular to the normal to the surface at \( p \).