We wish to evaluate
\[ \int \int_D e^{-(x^2+y^2)} \, dxdy. \]

Under the polar coordinate change of variables
\[ x = r \cos(\theta) \]
and
\[ y = r \sin(\theta), \]
the annular region \( D \) corresponds to the closed rectangle
\[ E = \{(r, \theta) : 1 \leq r \leq 3, 0 \leq \theta \leq \pi\}, \]
as illustrated in Figure 3.7.6. Moreover, \( x^2 + y^2 = r^2 \) and, as we saw in the previous example,
\[ \left| \det \frac{\partial(x,y)}{\partial(r,\theta)} \right| = r. \]

Hence
\[
\int \int_D e^{-(x^2+y^2)} \, dxdy = \int \int_E r e^{-r^2} \, drd\theta
= \int_1^3 \int_0^\pi r e^{-r^2} \, d\theta dr
= \int_1^3 \pi r e^{-r^2} \, dr
= \frac{\pi}{2} e^{-r^2} \bigg|_1^3
= \frac{\pi}{2} (e^{-1} - e^{-9}).
\]

Note that in this case the change of variables not only simplified the region of integration, but also put the function being integrated into a form to which we could apply the Fundamental Theorem of Calculus.