Some pictures of skew lines

Think of the solid rectangular box as the set of all points \((x, y, z)\) such that \(x\) lies between 0 and \(a\), \(y\) lies between 0 and \(b\), and \(z\) lies between 0 and \(c\), where \(a\), \(b\) and \(c\) are all positive real numbers. Then the line indicated by arrows in the bottom plane is the one joining the vertices \((0, b, 0)\) and \((a, 0, 0)\), while the line indicated by arrows in the top plane is the one joining the vertices \((0, 0, c)\) and \((a, b, c)\). To see that the two lines are skew lines, it is enough to show that there is no plane containing these four vertices. But if such a plane existed then the points would satisfy an equation \(Px+Qy+Rz = K\) for some \(P, Q, R, K\) where not all of the first three numbers are zero, and if we substitute the previous four points into this equation we get \(Qb=K=Pa=Rc=Pa+Qb+Rc\). If this system has a solutions then adding the first three equations yields \(K=Pa+Qb+Rc=3K\), which means that \(K = 0\); this and the original system imply that \(a, b, c\) must all be zero. Since we are assuming all three numbers are positive, it follows that no plane contains the given four points.

Note that the two lines have a common perpendicular which is a vertical line through the center of the rectangular solid.

Yet another example of the same type appears on the next page.
In this case the line on the lower face passes through \((0, 0, 0)\) and \((a, 0, 0)\), while the line on the upper face passes through \((a, 0, c)\) and \((a, b, c)\). An argument like the preceding one shows that these four points are not coplanar. The details of checking this out are left as an exercise. In this case, the common perpendicular is the vertical line passing through \((a, 0, c)\) and \((a, 0, 0)\).