The Tractrix Curve

The opening statement of the Wikipedia article (http://en.wikipedia.org/wiki/Tractrix) summarizes things very well:

[The] tractrix (from the Latin verb trahere “pull, drag”; plural: tractrices) is the curve along which an object moves, under the influence of friction, when pulled on a horizontal plane by a line segment attached to a tractor (pulling) point that moves at a right angle to the initial line between the object and the puller at an infinitesimal speed. It is therefore a curve of pursuit. It was first introduced by Claude Perrault in 1670, and later studied by Sir Isaac Newton (1676) and Christian Huygens (1692).

(Source: http://mathworld.wolfram.com/Tractrix.html)

There is an animated version of this drawing in the Wikipedia article. The following parametric equations are given in http://xahlee.org/SpecialPlaneCurves_dir/Tractrix_dir/tractrix.html:

\[
\left( \log(\sec t + \tan t) - \sin t, \cos t \right), \quad -\pi/2 < t < \pi/2.
\]

As indicated in Unit IV of the course notes, the surface of revolution obtained by rotating the tractrix around the \(x\) – axis, which is called the pseudosphere, has constant negative Gaussian curvature.

FOOTNOTE. Since the Riemannian metric for the classical non – Euclidean (or hyperbolic) plane also has constant negative Gaussian curvature, it is useful to explain the relationship between hyperbolic and pseudospherical geometry. Although the geometry of the pseudosphere is not the same as classical non – Euclidean (or hyperbolic) geometry, suitably chosen small pieces of each turn out to have the same intrinsic geometrical properties (in the same sense that small pieces of a plane and cylinder have the same intrinsic geometrical properties).