Applied Category Theory: A Philosophical Introduction

David Corfield

University of Kent

6 June, 2023

Yale announces

Experimental Philosophy
@xphilosopher

Yale philosophy has officially replaced the grad program “logic requirement” with a broader “formal methods requirement.”

Students can choose which course to take (logic, probability, stats, game theory, etc.)

Feels like a symptom of a much larger change in the discipline

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My advice to Yale

David Corfield
@DavidCorfield8

So teach them category theory as core + optional add-ons.

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Formalisms encountered in a philosophical training

- **Likely**: Propositional logic, first-order logic, modal logic.
- **Possible**: Second-order logic, probability theory, decision theory, set theory, ...
- **Unlikely**: Type theory,...
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Note: There are different kinds of exposure. Here I'm more interested in the ways of thinking afforded by the system.

Applications of category theory

(With approximate dates)

- Mathematics (from the 1940s)
- Logic/Foundations (from the 1960s)
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Category Theory	Physics	Topology	Logic	Computation
object X	Hilbert space X	manifold X	proposition X	data type X
morphism $f\colon X \to Y$	operator $f\colon X \to Y$	cobordism $f\colon X \to Y$	proof $f\colon X \to Y$	program $f\colon X \to Y$
tensor product of objects: $X \otimes Y$	Hilbert space of joint system: $X \otimes Y$	disjoint union of manifolds: $X \otimes Y$	conjunction of propositions: $X \otimes Y$	product of data types: $X \otimes Y$
tensor product of morphisms: $f \otimes g$	parallel processes: $f \otimes g$	disjoint union of cobordisms: $f \otimes g$	proofs carried out in parallel: $f \otimes g$	programs executing in parallel: $f \otimes g$
internal hom: $X \multimap Y$	Hilbert space of 'anti-X and Y': $X^* \otimes Y$	disjoint union of orientation-reversed X and Y: $X^* \otimes Y$	conditional proposition: $X \multimap Y$	function type: $X \multimap Y$

Table 4: The Rosetta Stone (larger version)

(Baez and Stay 2009, Physics, Topology, Logic and Computation: A Rosetta Stone)

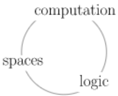
Computational Trinitarinism/Trilogy

1. Idea

A profound cross-disciplinary insight has emerged – starting in the late 1970s, with core refinements in recent years – observing that three superficially different-looking fields of mathematics,

- computation/programming languages
- formal logic/type theory
- ∞-category theory/∞-topos theory (algebraic topology)

are but three different perspectives on a single underlying phenomenon at the foundations of mathematics:



(nLab: computational trilogy)

Good things continue to happen interrelating these subjects, as I report in:

- *Modal Homotopy Type Theory: The prospect of a new logic for philosophy*, (OUP 2020)
- *Thomas Kuhn, Modern Mathematics and the Dynamics of Reason*, PhilSci

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But today we have a different focus, though very much one in the same universe of ideas.

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  Causality, probabilistic reasoning, statistics, learning theory, deep neural networks, dynamical systems, information theory, database theory, natural language processing, cognition, consciousness, systems biology, genomics, epidemiology, chemical reaction networks, neuroscience, complex networks, game theory, robotics, quantum computing,...

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A key term is Compositionality.

- Plug together systems in parallel.
- Plug together systems in series.
- Plug one system inside another.

Today's talks

- Toby St Clere Smithe
- John Baez

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Toby's DPhil: *Mathematical Foundations for a Compositional Account of the Bayesian Brain*, arXiv:2212.12538

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How to unify all my academic interests: (i) category theory, type theory, machine learning, Bayesianism, (ii) psychoanalysis, psychosomatic medicine, and maybe even (iii) the dynamic development of scientific/mathematical theory.

Typing

Our primary motivation in writing this thesis is to lay the groundwork for *well-typed* cognitive science and computational neuroscience. 'Types' are what render categorical concepts so precise, and what allow categorical models to be so cleanly compositional: two systems can only "plug together" if their interface types match. Because every concept in category theory has a type (*i.e.*, every object is an object of some category), categorical thinking is forced to be very clear. As we will sketch in §2.3.4, the "type theories" (or "internal languages") of categories can be very richly structured, but still the requirement to express concepts with types is necessarily burdensome. But this burden is only the burden of thinking clearly: if one is not able to supply a detailed type, one can resort to abstraction. And, to avoid the violence of declaring some object to be identified as of some type[7], it is necessary to understand the relationships between types; fortunately, as we will soon make clear, and as we have attempted to emphasize, category theory is fundamentally the mathematics of relationship.