Formal composition of hybrid systems

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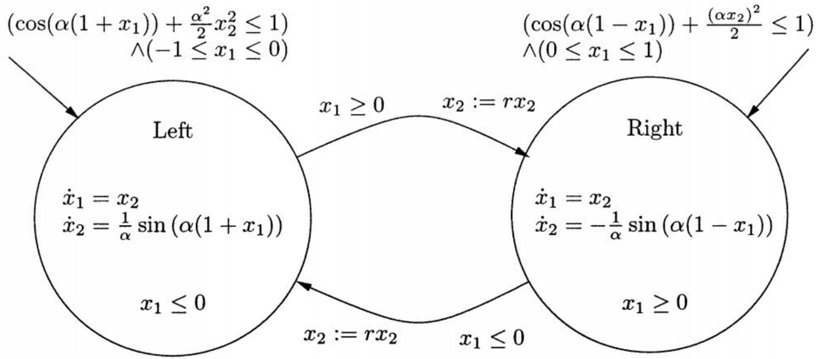
November 2019

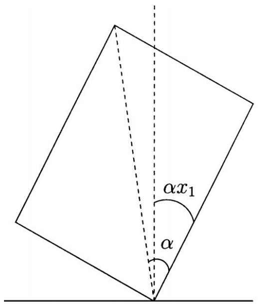
Composition of Robotic Behaviors

Hybrid systems

A **hybrid system** H consists of

- a directed graph $G = (V, E, \mathfrak{s}, \mathfrak{t})$;
- for each **mode** $v \in V$,
  - an **ambient smooth system** (M_v, X_v)
  - an **active set** $I_v \subset M_v$
  - a **flow set** $F_v \subset I_v$
- for each **reset** $e \in E$, a **guard set** $Z_e \subset I_{\mathfrak{s}(e)}$ and an associated **reset map** $r_e \colon Z_e \to I_{\mathfrak{t}(e)}$.





Morphisms: hybrid semiconjugacies

- "execution-preserving maps"

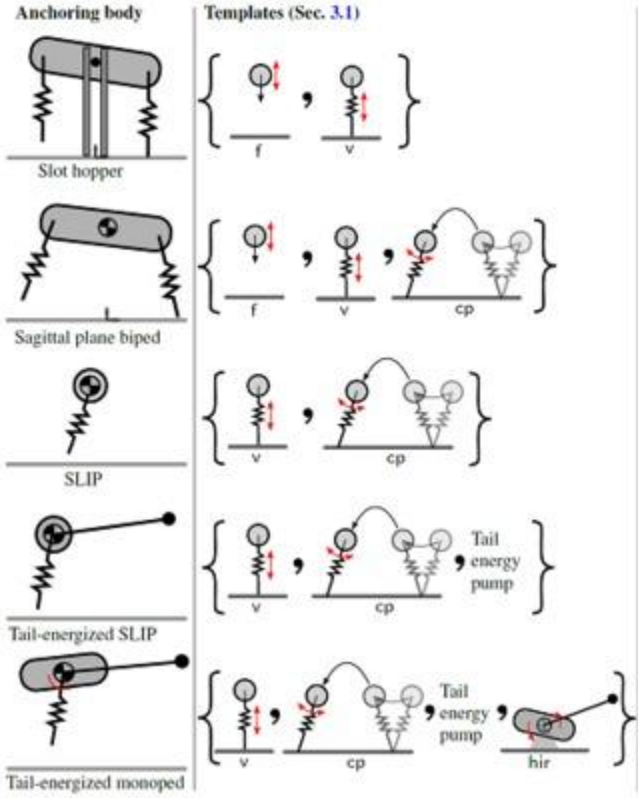
Cf. Lerman. "A category of hybrid systems." arXiv:1612.01950.

Image source: Lygeros et al., "Dynamical properties of hybrid automata." IEEE Transactions on automatic control, 2003.

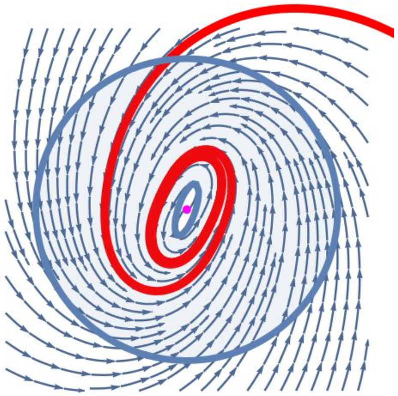
Templates and anchors

A **template-anchor pair** is a span $T \overset{p}{\leftarrow} S \overset{i}{\rightarrow} A$ such that

- p is a hybrid subdivision;
- i is a hybrid embedding;
- $i(S)$ is an isolated invariant set in A.



De, Avik, and Daniel E. Koditschek. "Parallel composition of templates for tail-energized planar hopping." 2015 IEEE International Conference on Robotics and Automation (ICRA). IEEE, 2015.



A classical limit cycle with isolating annulus in blue

Subdivisions of hybrid systems

A **hybrid subdivision** a hybrid submersion $p \colon S \to H$ such that for every hybrid time execution $\chi \colon \tau \to H$, there exists a pullback square

$$\begin{array}{ccc} \sigma & \xrightarrow{\widetilde{\chi}} & S \\ {\scriptstyle \xi}\downarrow & \lrcorner & \downarrow{\scriptstyle p} \\ \tau & \xrightarrow[\chi]{} & H \end{array}$$

such that ξ is a **refinement** of hybrid time trajectories.

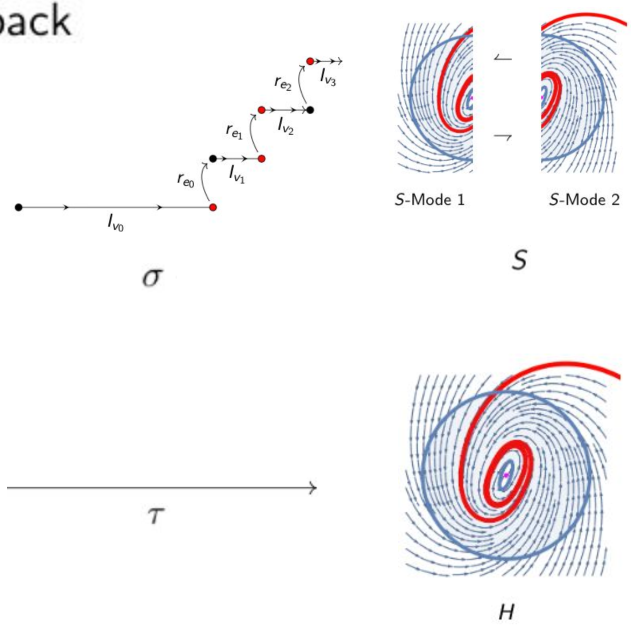
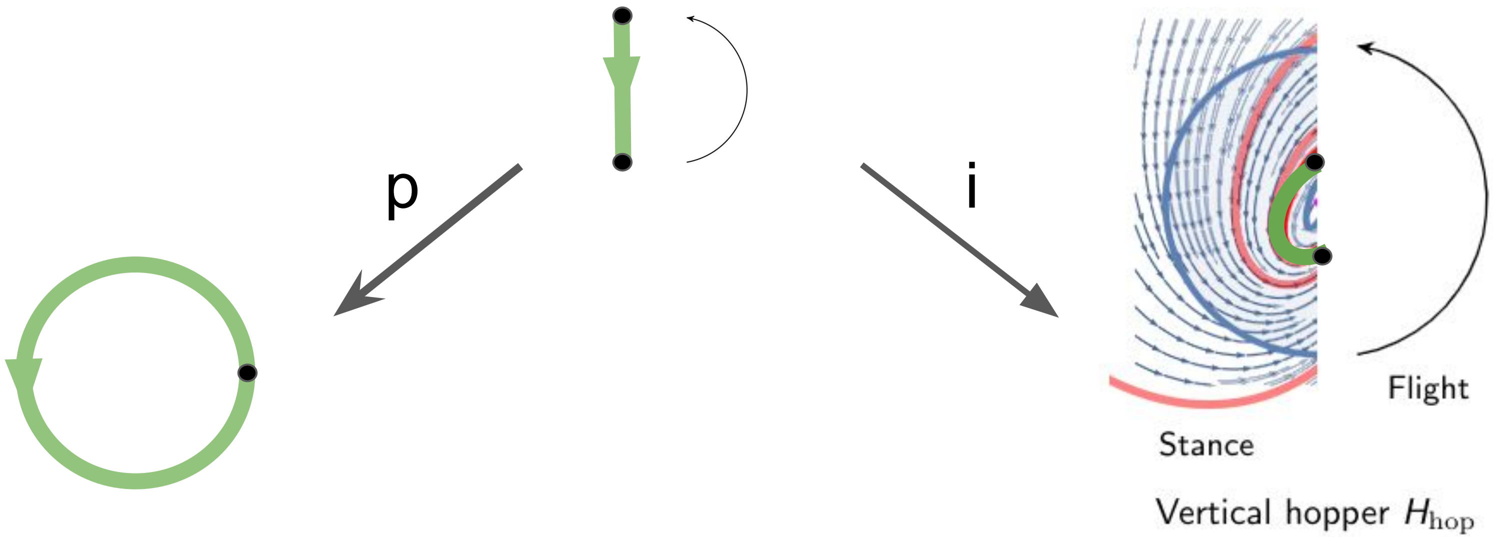


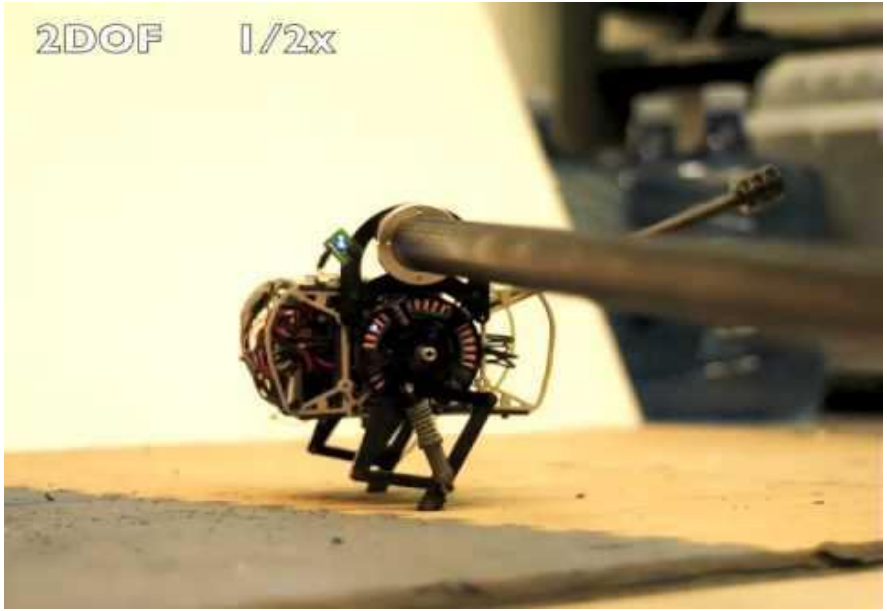
Image source: De and Koditschek. "Parallel composition of templates for tail-energized planar hopping." 2015 IEEE International Conference on Robotics and Automation.

Anchoring a limit cycle in a vertical hopper



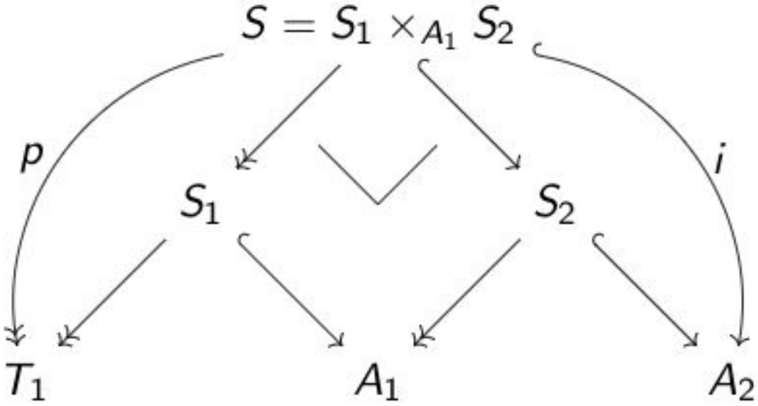
De and Koditschek. "Parallel composition of templates for tail-energized planar hopping." 2015 IEEE International Conference on Robotics and Automation.

Hierarchical composition

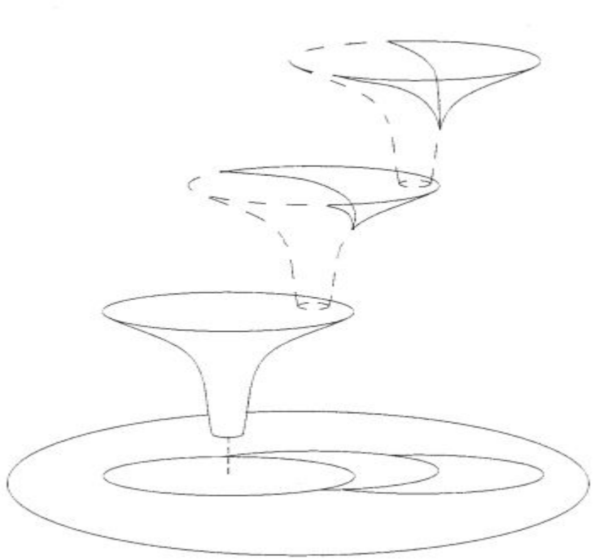


De, Avik, and Daniel E. Koditschek. "Parallel composition of templates for tail-energized planar hopping." 2015 IEEE International Conference on Robotics and Automation (ICRA). IEEE, 2015.

Theorem (CGKS). Template-anchor pairs are weakly associatively composable.



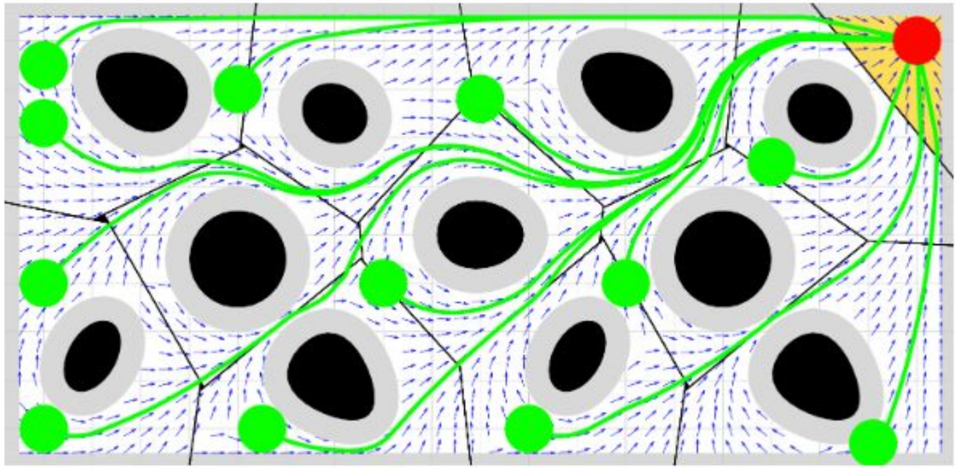
Sequential composition



Burridge, Robert R., Alfred A. Rizzi, and Daniel E. Koditschek. "Sequential composition of dynamically dexterous robot behaviors." *The International Journal of Robotics Research* 18.6 (1999): 534-555.

Goal: define “funnel-like” sequentially composable hybrid systems

A “navigate-to-goal” funnel



Theorem 3. *The piecewise continuously differentiable “move-to-projected-goal” law in (11) leaves the robot’s free space* $\mathcal{F}$ *(1) positively invariant; and if Assumption 2 holds, then its unique continuously differentiable flow, starting at almost[1] any configuration* $\mathrm{x} \in \mathcal{F}$*, asymptotically reaches the goal location* x^**, while strictly decreasing the squared Euclidean distance to the goal,* $\|\mathrm{x} - \mathrm{x}^*\|^2$*, along the way.*

Arslan, Omur, and Daniel E. Koditschek. "Sensor-based reactive navigation in unknown convex sphere worlds." *The International Journal of Robotics Research* (2019).

How to define ``funnel-like'' systems?

- **Problem:** the naive measure-theoretic and topologically notions of "almost all" are incompatible with fully general sequential composition
- Example:

$$H = \overset{([-1,0],\frac{d}{dx})}{\bullet} \underset{0\mapsto 0}{\rightarrow} \overset{(\{0\},0)}{\bullet} \qquad K = \overset{([0,1],x\frac{d}{dx})}{\bullet} \underset{1\mapsto 1}{\rightarrow} \overset{(\{1\},0)}{\bullet}$$

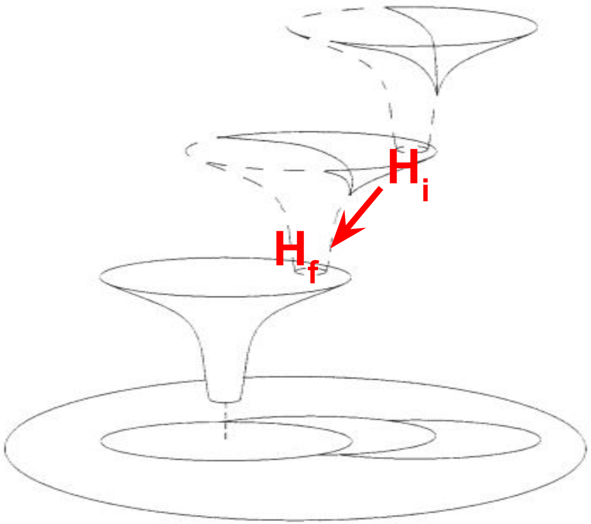
- Is there a notion of "generalized execution" compatible with sequential composition?

Directed systems

A **directed hybrid system** $H \colon H_i \rightsquigarrow H_f$ is a tuple (H, η_i, η_f) consisting of

- a metric hybrid system H,
- embeddings $\eta_i \colon H_i \to H$ and
- a hybrid embedding $\eta_f \colon H_f \to H$ such that each component $(\eta_f)_v$ is a diffeomorphism, and $G(H_f)$ is a sink in $G(H)$

such that for all $\varepsilon, T > 0$ and $x \in H$, there exists an (ε, T)-**chain** from x to some $y \in H_f$.



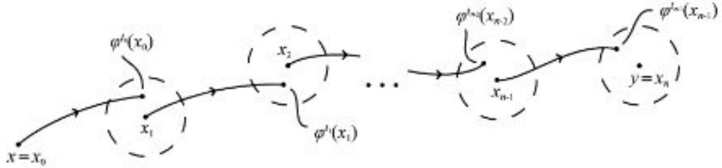
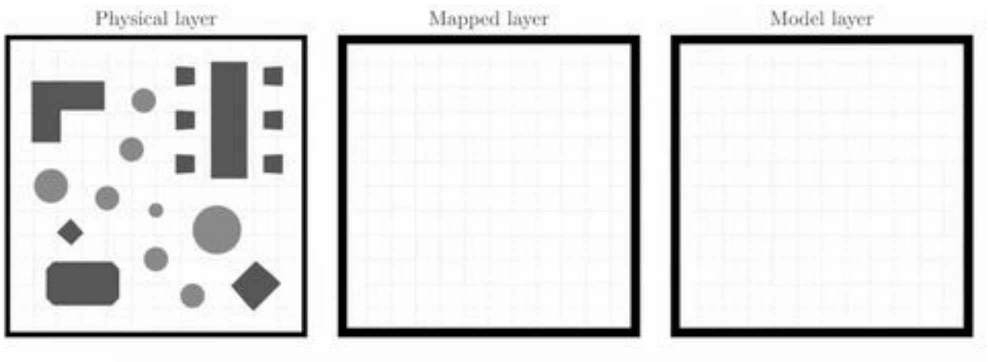
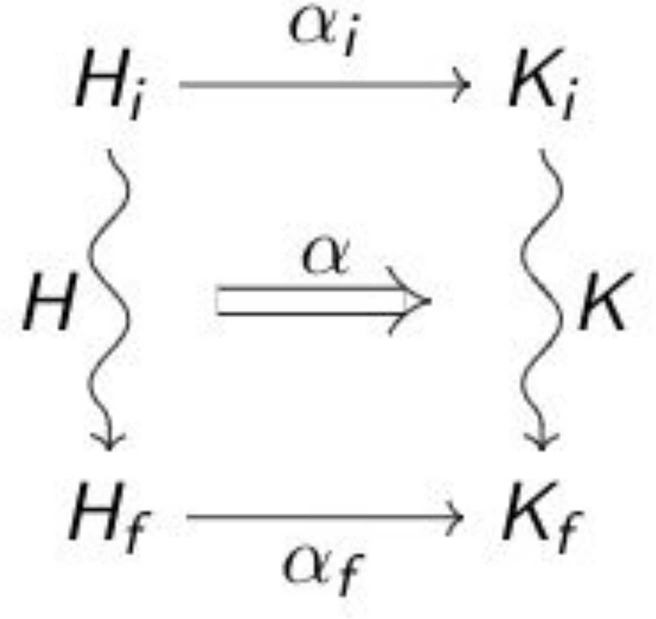


Image source: Alongi and Nelson, *Recurrence and Topology*. AMS, 2007.

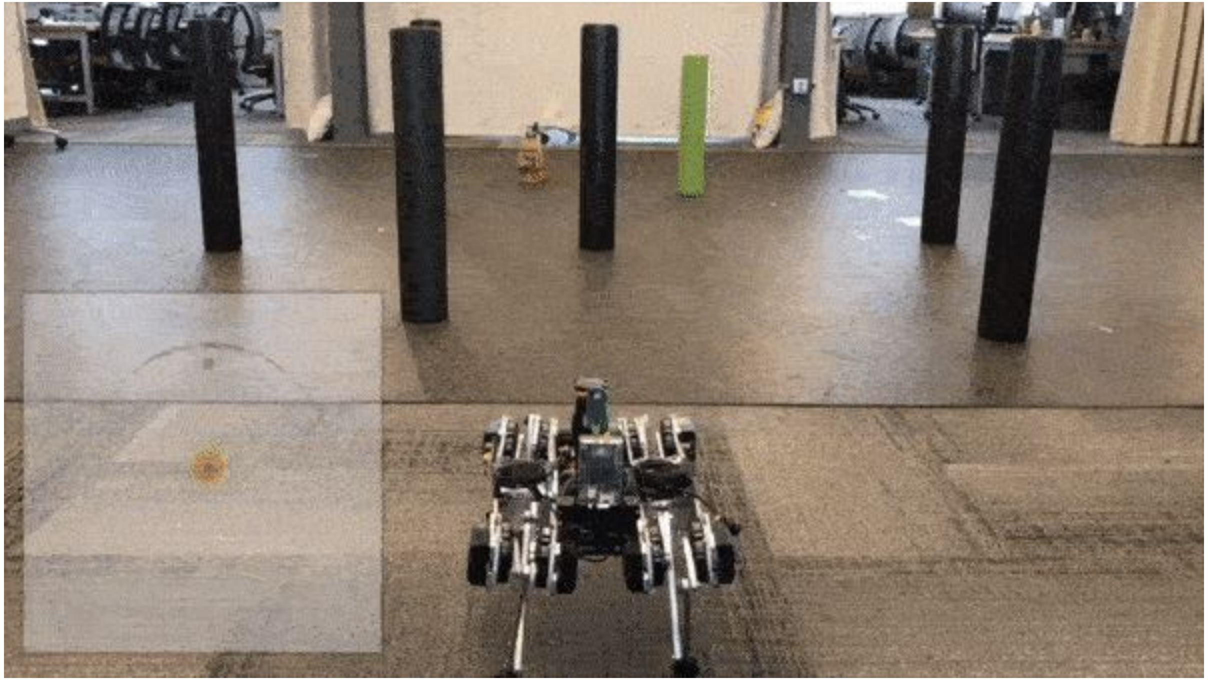
A double category of hybrid systems



V. Vasilopoulos, D.E. Koditschek (2018). Reactive Navigation in Partially Known Non-Convex Environments. In WAFR 2018.



Reactive Navigation in Non-Convex Environments



V. Vasilopoulos, D.E. Koditschek (2018). Reactive Navigation in Partially Known Non-Convex Environments. In WAFR 2018.

Further Directions

- **Internal languages of double categories**
  - New and Licata. "Call-by-name gradual type theory." arXiv:1802.00061 (2018).
- **Connection to LTL/FRP**
  - Kress-Gazit, Fainekos, and Pappas. "Temporal-logic-based reactive mission and motion planning." *IEEE transactions on robotics* (2009).
- **Compatibility with coupled parallel compositions**
  - Lerman and Schmidt. “Networks of hybrid open systems.” arXiv:1908.10447 (2019).
- **Triple categories**
  - Grandis and Paré. "Intercategories: a framework for three-dimensional category theory." *Journal of Pure and Applied Algebra* (2017).
- **Hybrid Conley theory**
  - Kvalheim, Gustafson, and Koditschek. “A hybrid version of Conley’s fundamental theorem of dynamical systems,” in preparation.

Thanks!

arXiv:1911.01267