Potential applications of category theory to design: Example of brakes

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Design of brakes

The purpose of the talk to is illustrate the process of design of brakes using CT as a potential modeling language

- Brakes are used in a large number of contexts.
- We will concentrate on brakes for car(t)s
- Brakes have evolved over time from simple devices to complex multi-technology devices

Requirements for a Brake

What is a requirement?

- External environment for brakes requirement:
  - The weight of the vehicle (W)
  - Material of the wheels (Mat - Coefficient of friction) (V)
  - Driving surface material (coefficient for Friction) (SD)
  - Specified speed of the Vehicle for stopping conditions:
  - Stopping distance at a given velocity: 20 feet at 20 miles/hour

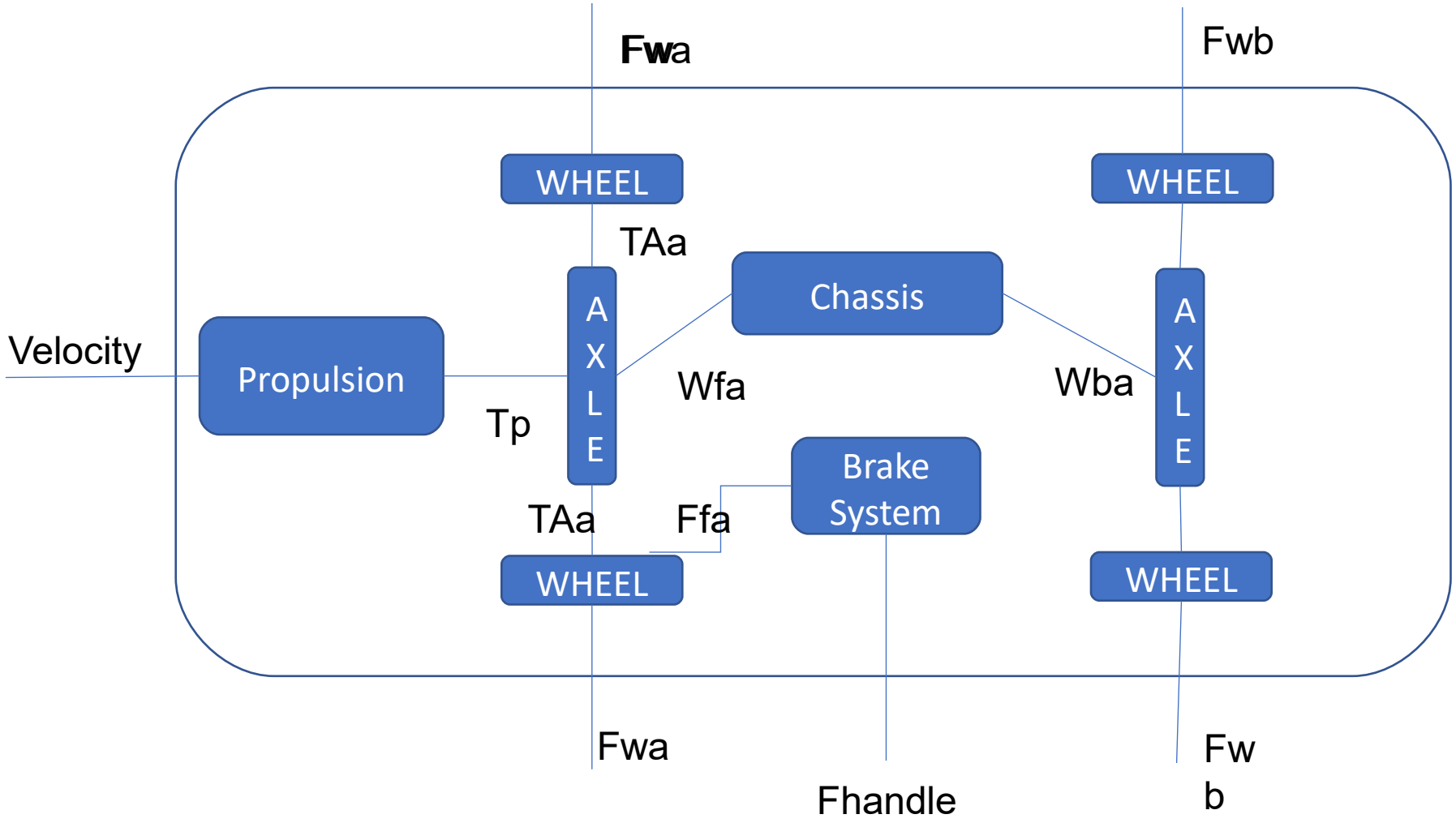
What is a requirement for the design? $(W \times V \times SD)$ $(R \subseteq W \times V \times SD)$

  - (implicit) State space
  - Subset of acceptable states (requirement)

Simplified design process

- Requirements
- Architecture
- Sketch/geometry
- Modeling
- Manufacturing
- Qualification and Testing

Architecture



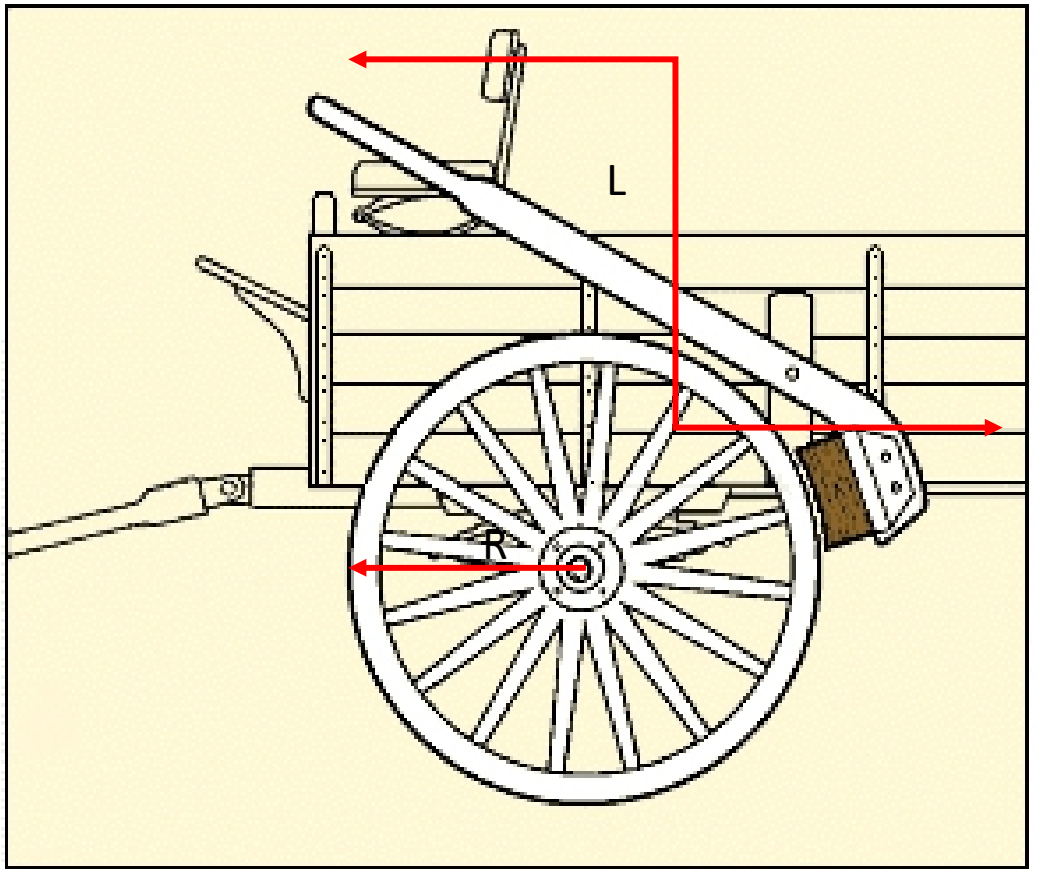
- Port graph/Wiring Diagram - Nodes are typed interfaces

Sketch

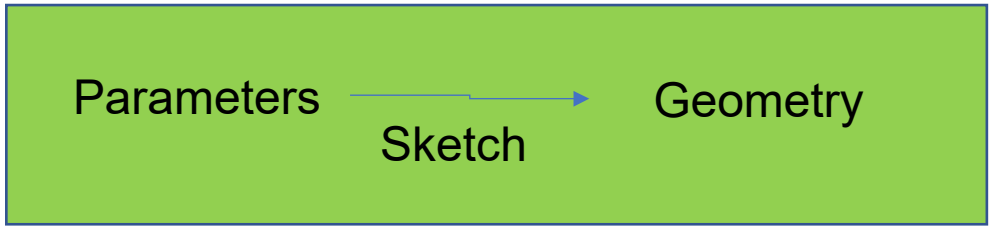
Involves

- Geometric elements
- Parameters

Can be represented as a Function



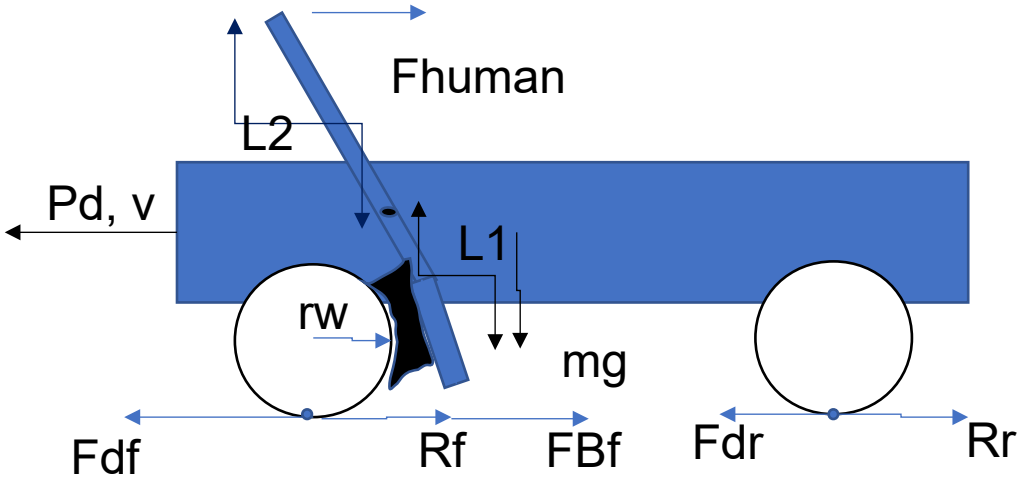
The wooden block and lever

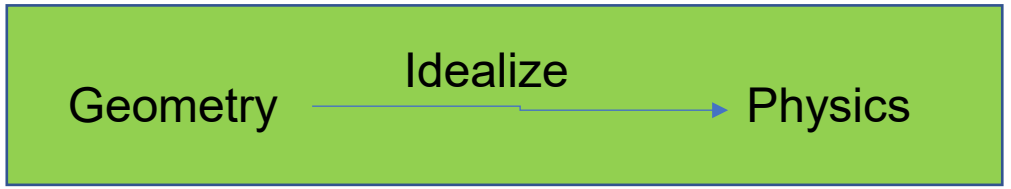


Sketch is now rendered as a CAD model (includes Materials, etc)

Idealization

- Simplification
  - Negligible lateral load transfer
  - Negligible longitudinal load transfer
  - Negligible significant *roll and pitch motion*
  - …….
- Parameters
  - Radius of the wheel rw
  - Velocity (*V*)= 20 mph
  - Power Pd= 5Hp
  - Free body diagrams (forces)





Solve and Simulate

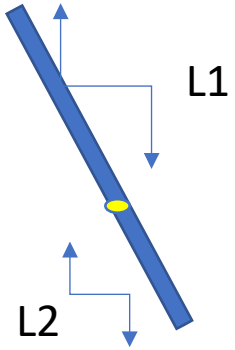
- Equations

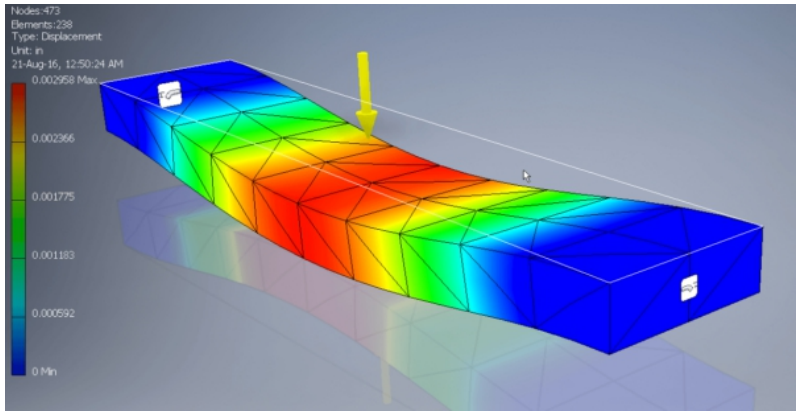
  Force at Brake

  - Rr = Rrf + Rrb = Cf mg (-v)
  - Fbf = PowerD/V – Cfmg (v)

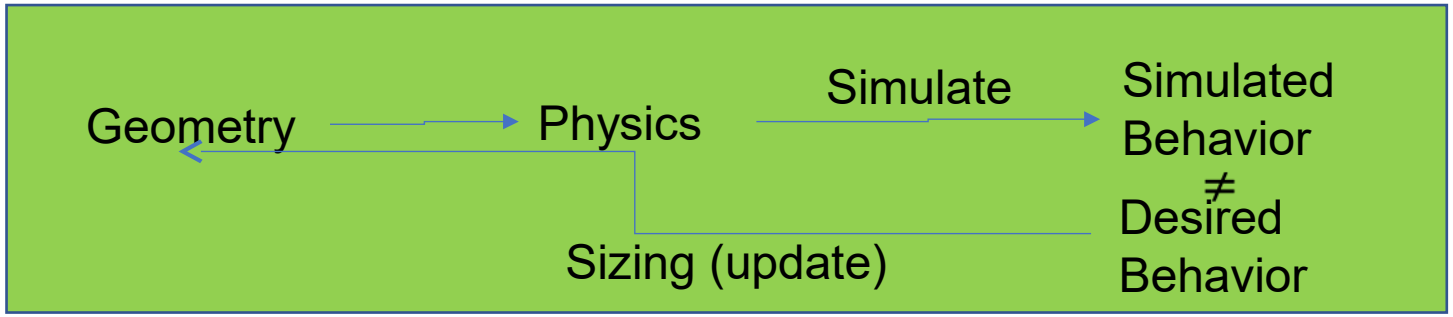
  Pivot location

  - L2/L1 = (Powerd - Cfmgv2)/ Fhuman

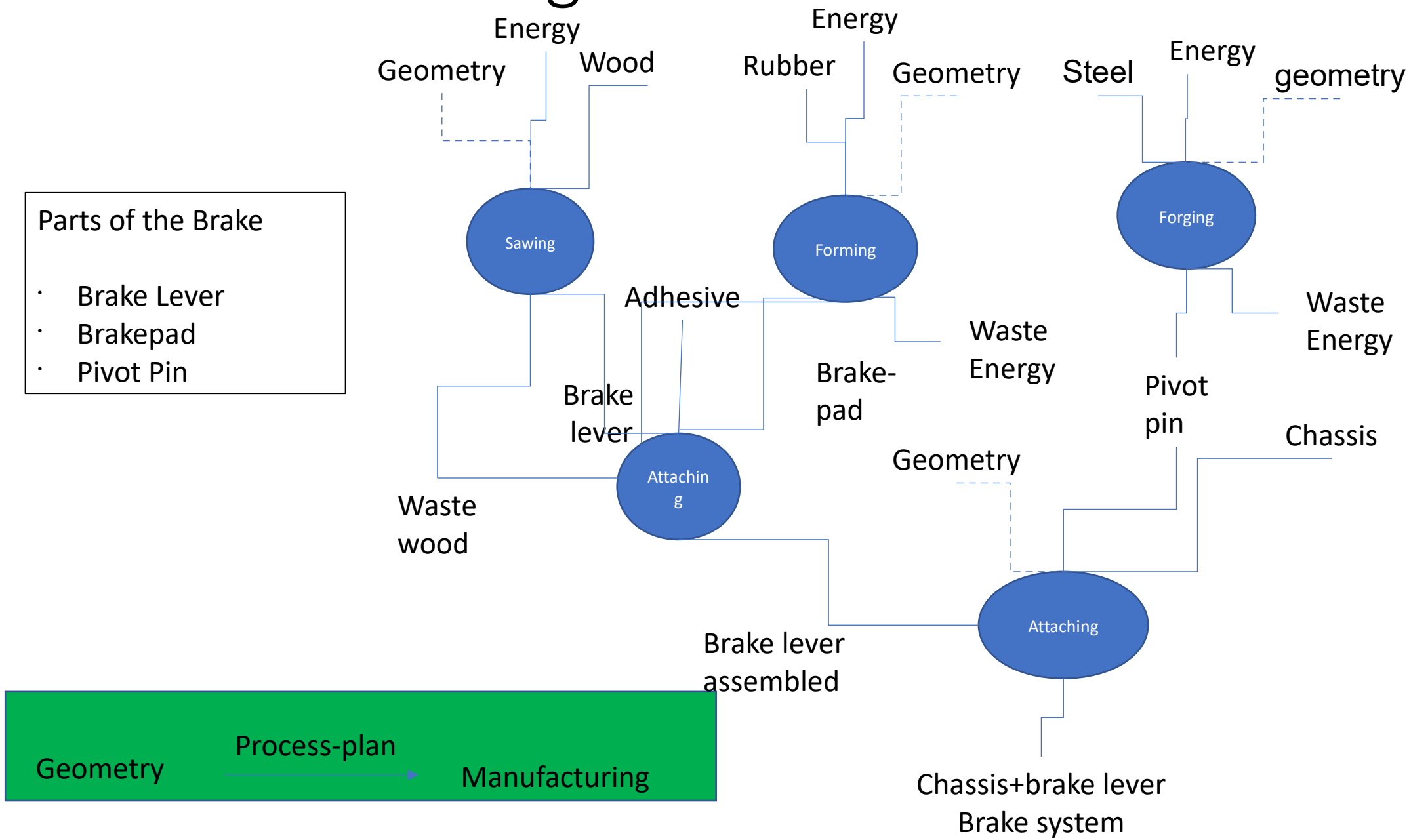




- Iteratively determine
  - Fbr
  - Dimensions of the Lever



Manufacturing Process Plan



Post-Manufacturing

Geometry as measured

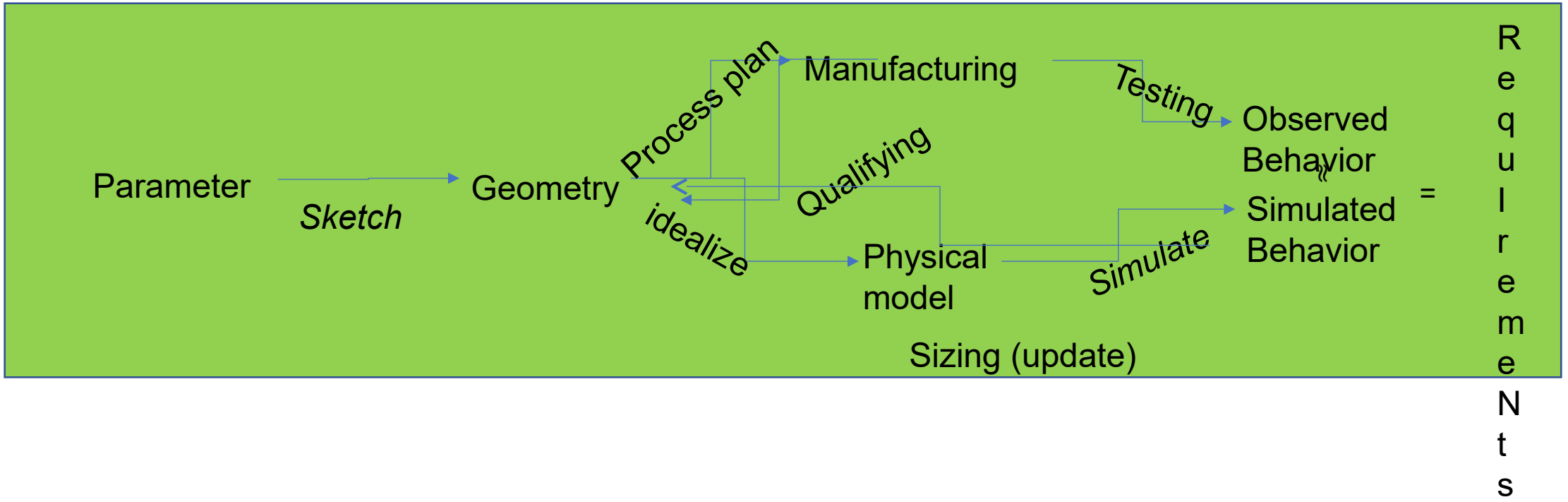
Qualifying

Manufacturing

Testing

Observed Behavior

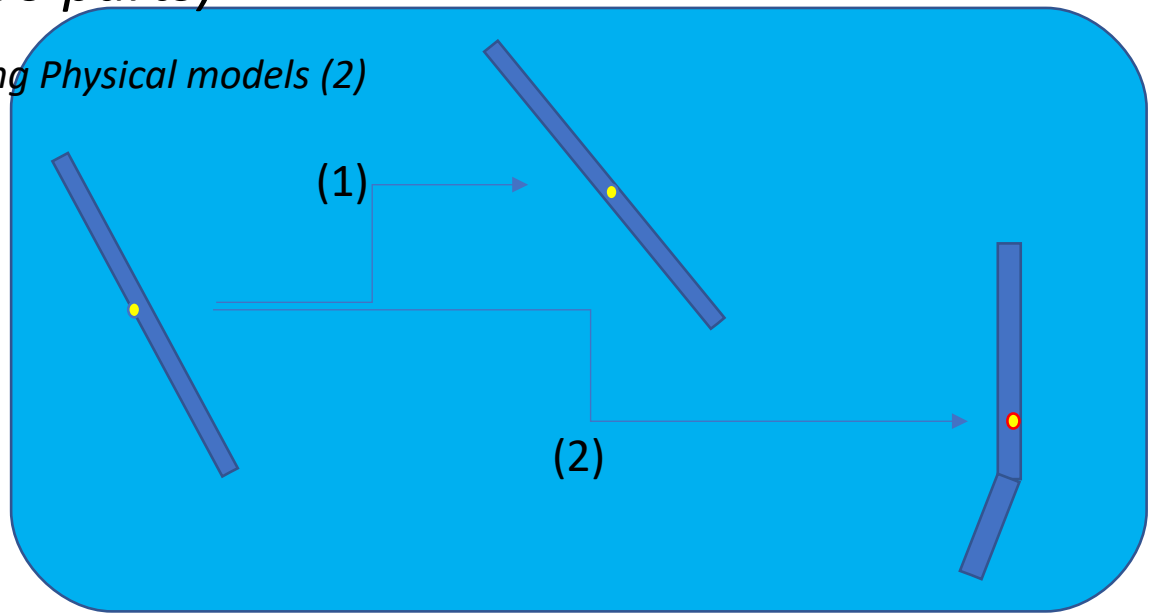
Product design and manufacturing process



Design Feedback Loops

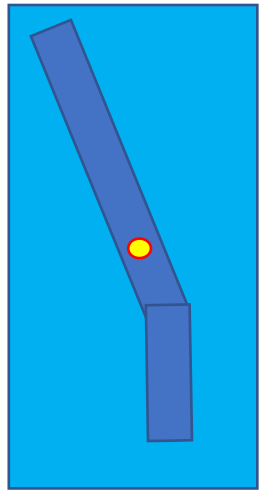
Physics based models do not meet requirements

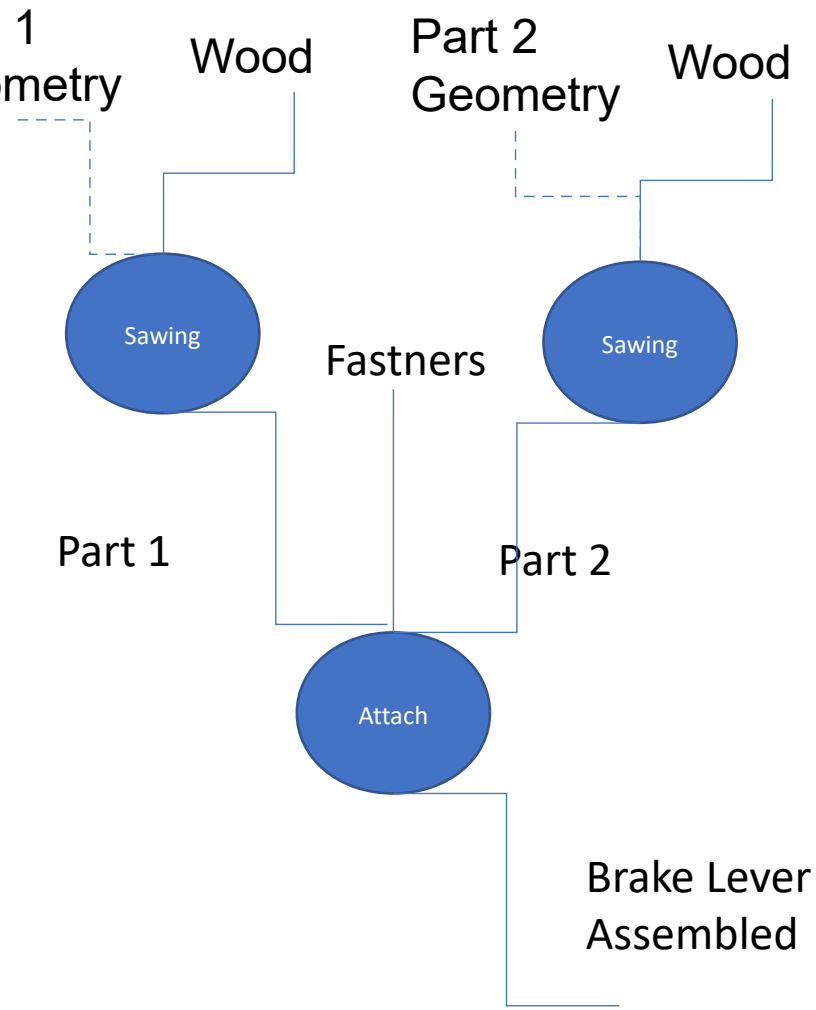
- Physics based models don’t meet requirements
  - Change the *specification (1)*
    - Change Parameter Value (example: Move Pivot Point)
  - Change the *geometry (two parts)*
    - *Change parameters and Mapping Physical models (2)*
  - *Opt*imization
    - Similar to above for shape



Changed geometry → Change manufacturing

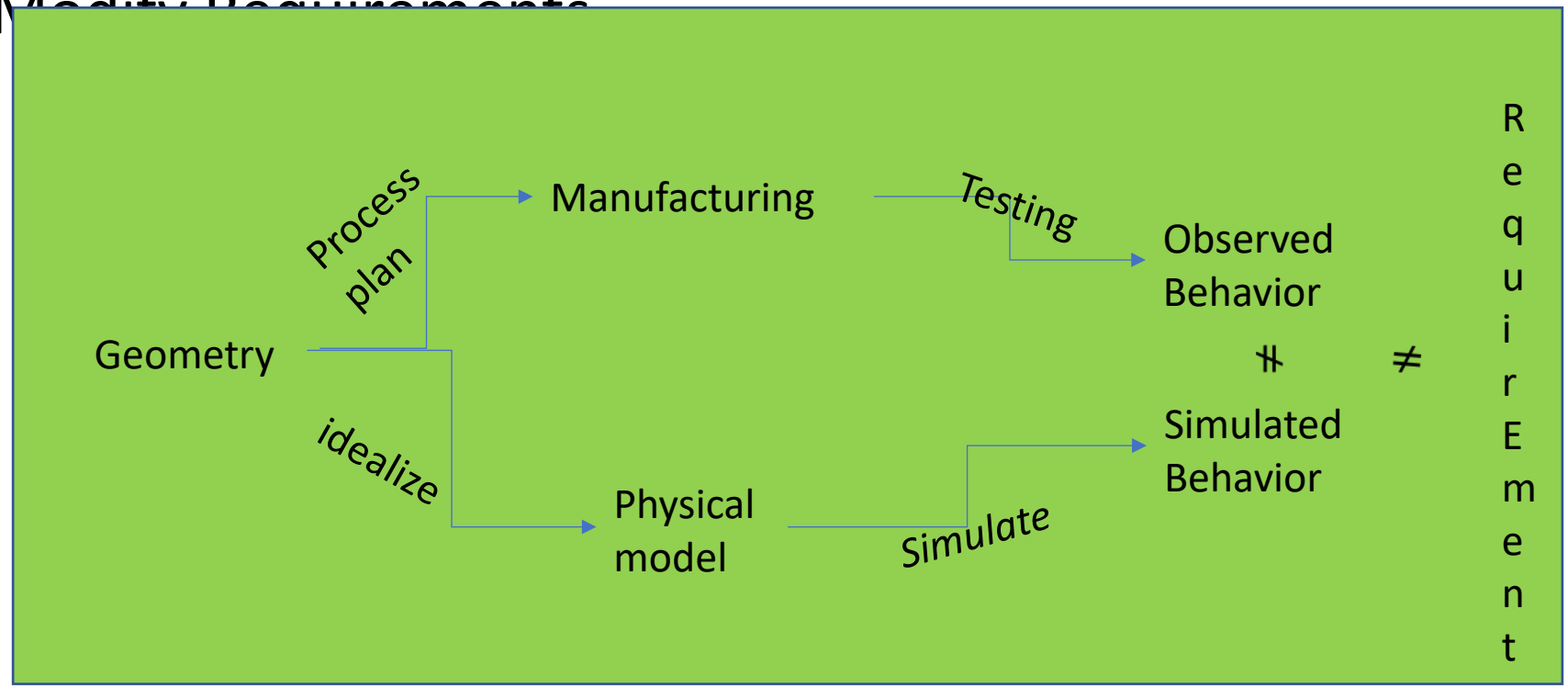
- Changed part geometry
  - Change process plan





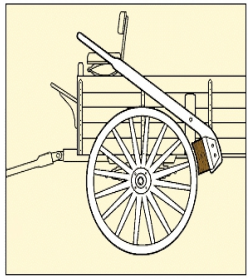
Physics model and testing diverge

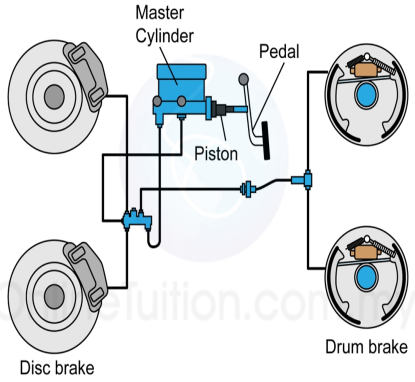
- Physics model and testing diverge
  - Tune parameters (change in the Simulation)
  - More complex model (change “Idealize”)
- Modify Requirements

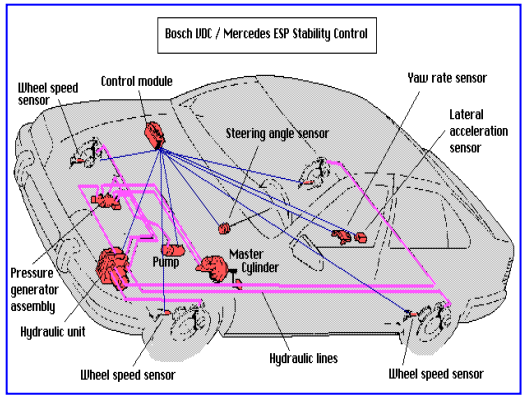


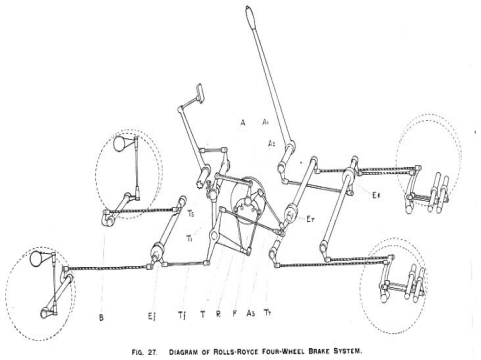
Product Evolution
(change in requirement and architecture)

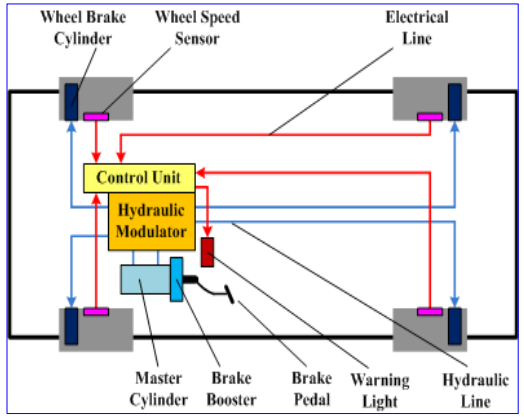
Evolution of Brake systems: change in amplification and control

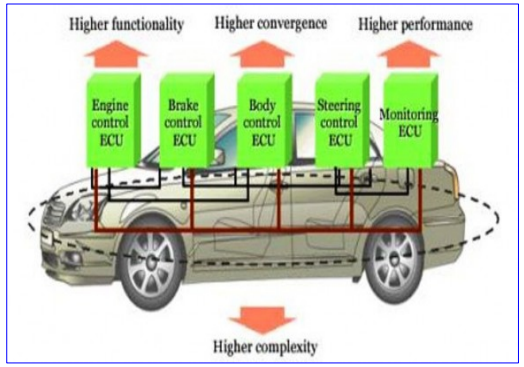






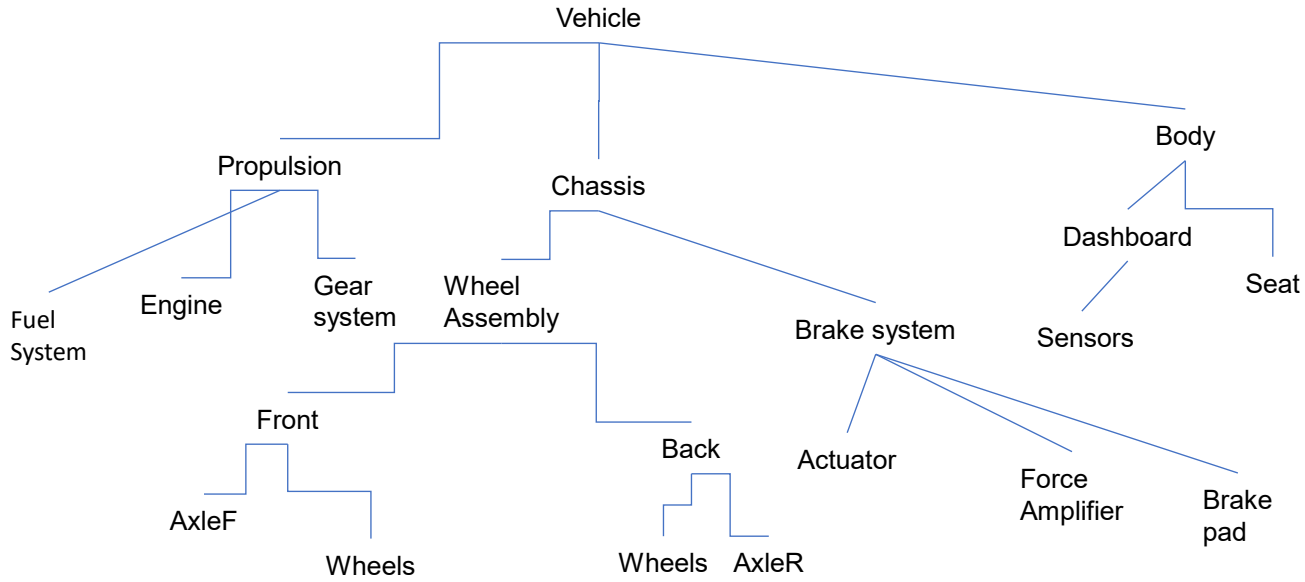


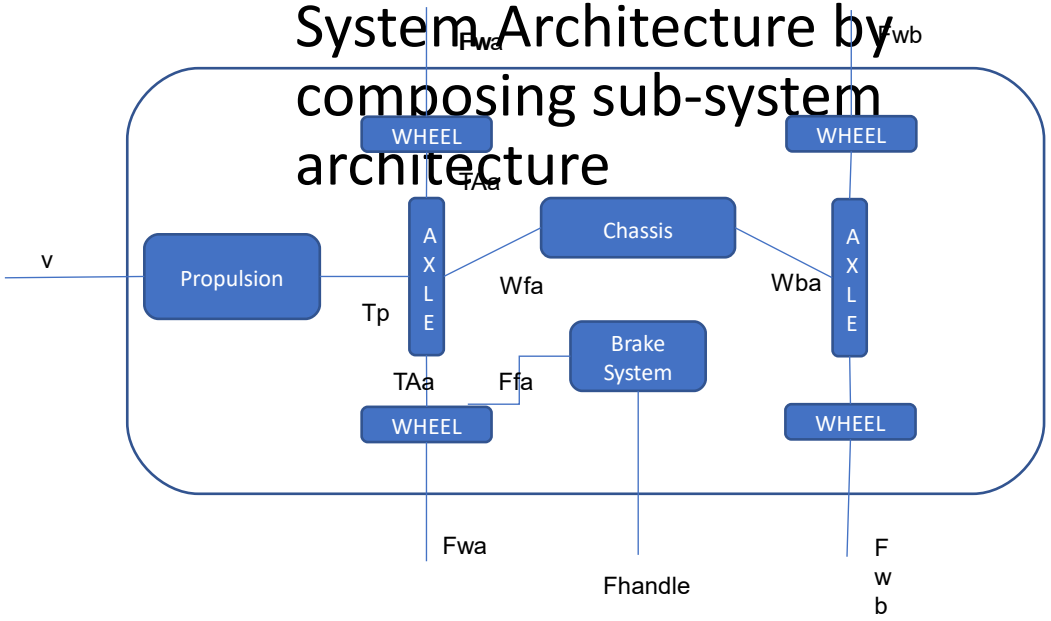


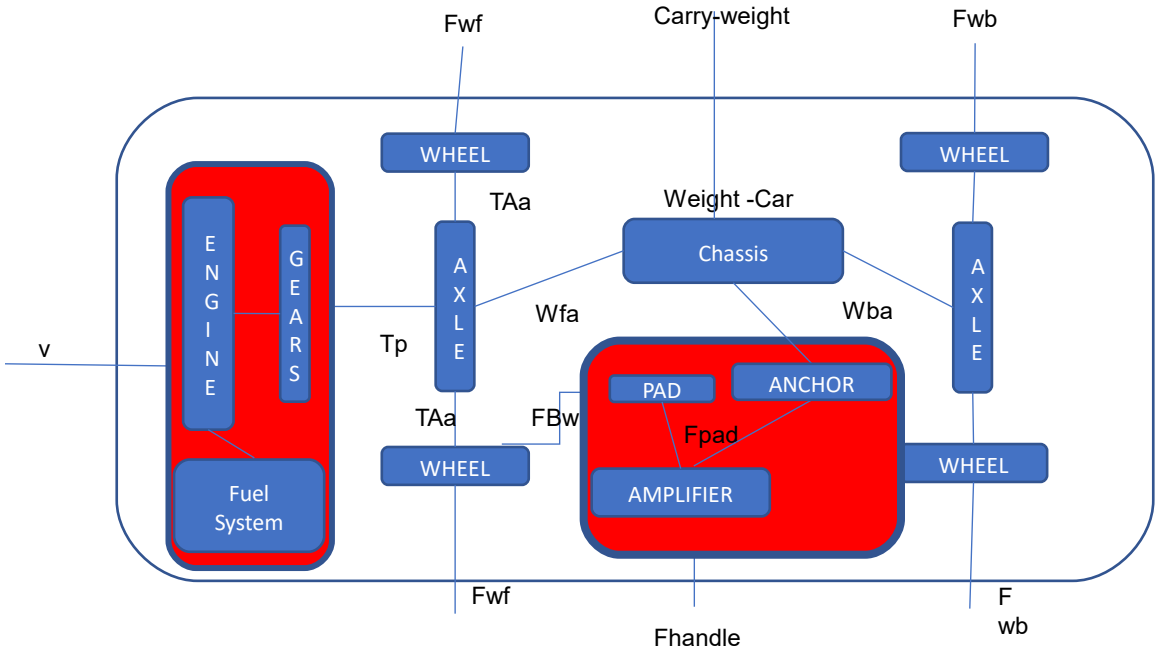


Modularity and Composition

- Port Graphs form the arrows of Operad
  - Operad is a category with tree shaped composition
  - Subsystem/part decomposition defines System Architecture by composing sub-system architecture







Everything is an operad in the product development Process

- Parameters: Aggregate across branches
  - Specification for parts gives the specifications for the whole
- Geometry
  - Geometry of the vehicle is the union of the geometry of all parts and subsystems
- Physics models
  - ??? Open area for research
- Manufacturing
  - Operad of String diagrams
- Requirement
  - Morphism $\phi: x_1, \ldots, x_k \to Y$ is a Wiring diagram operad then

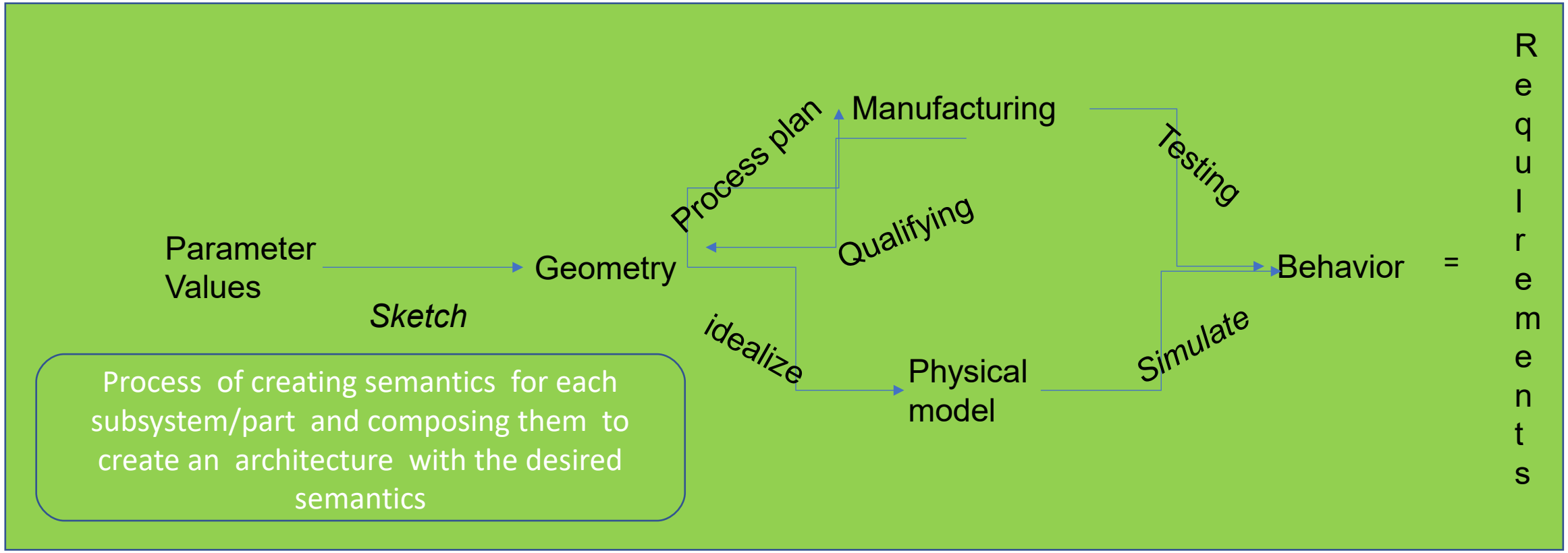
    $\text{Req}(\Phi)$: $\prod \text{Req}(x_i) \to \text{Req}(Y)$

For each morphism in the Architecture we need to go through the whole process, to align the models with the real world test observation

Req(Req(Req(Y)

For each morphism in the Architecture we need to go through the whole process, to align the models with the real world test observation

Architecture is syntax and the rest are semantics



References and Future work

String diagrams for process planning

- Spencer Briener, Albert Jones and Eswaran Subrahmanian, Categorical models for process planning, Computers and Industry, November, 2019.

Operads for architecture description and diagnosis

- Spencer Breiner, Olivier Marie-Rose, Blake S. Pollard, and Eswaran Subrahmanian, Operadic diagnosis in hierarchical systems, ACT Oxford

Future work

Thank you

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This work is done with Spencer Breiner and Blake Pollard at NIST.