Network Models from Petri Nets with Catalysts

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Quantum Physics and Logic
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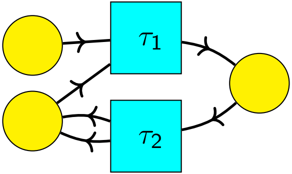
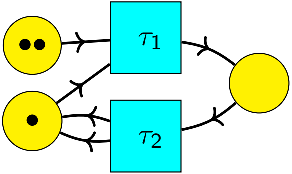
Petri Nets

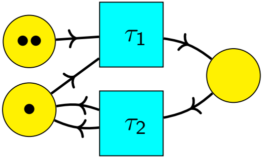


Markings



$$2A + 1B$$

Executions



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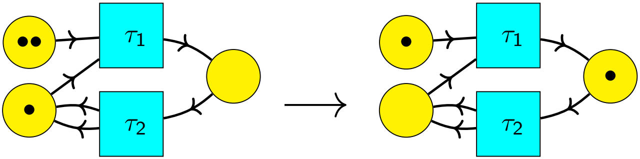

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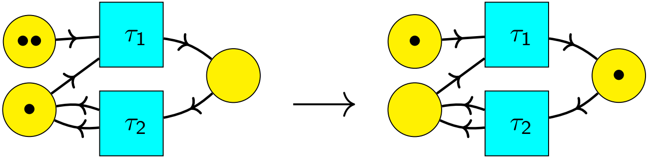
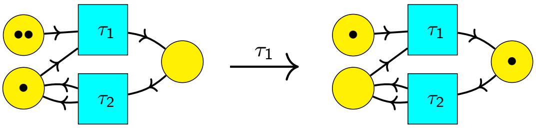
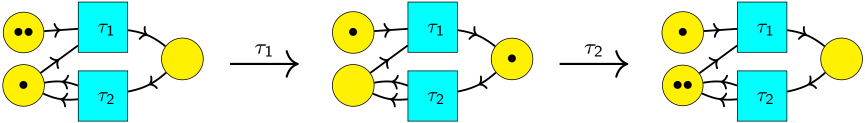


$$2A + 1B \xrightarrow{\tau_1} 1A + 1C$$

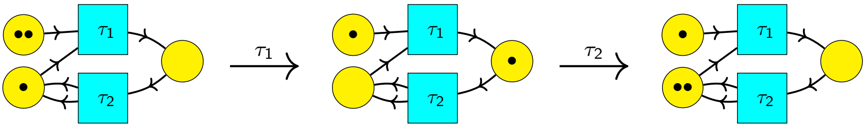
Sequential Execution



Sequential Execution

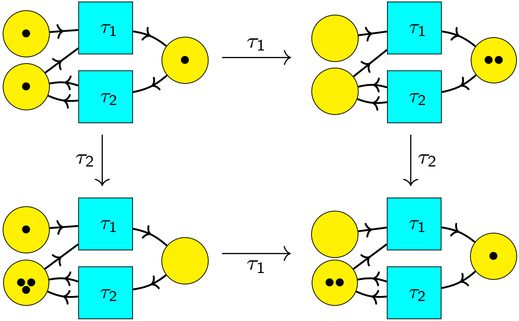


Sequential Execution

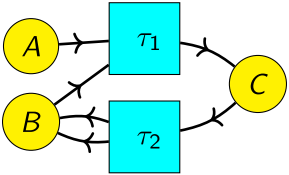


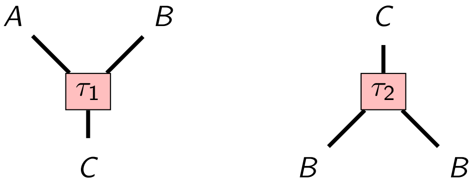
$$2A + 1B \xrightarrow{\tau_1} 1A + 1C \xrightarrow{\tau_2} 1A + 2B$$

Concurrent Execution



String Diagrams





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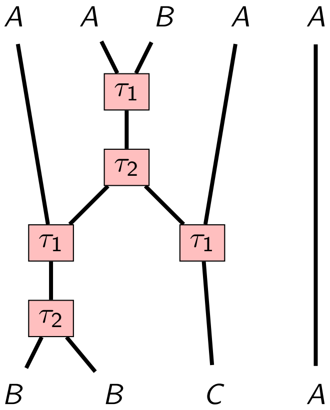
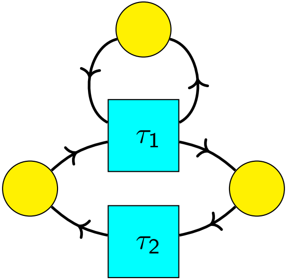


Catalysts



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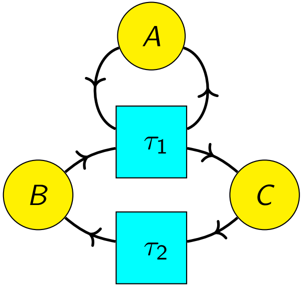
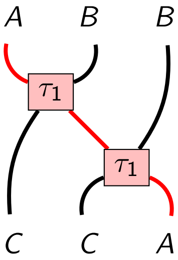
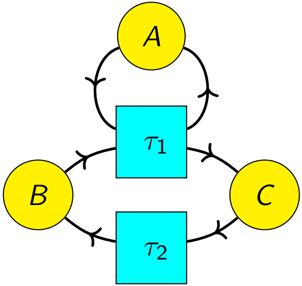




Catalysts



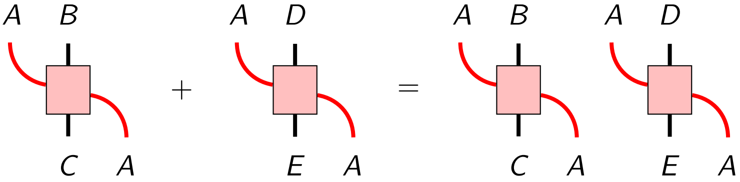
$A \quad B \quad B$

$C \quad A \quad A$

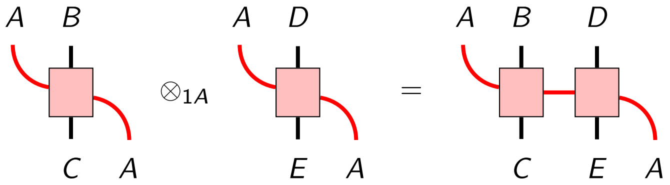
FP is a coproduct

$$FP = \coprod_{c \in \text{Catalysts}} FP_c$$

Not monoidal subcategories



Premonoidal Structure on Subcategories



Monoidal Grothendieck Construction

Theorem (Vasilakopoulou, M.)

If $\mathcal{X}$ is cocartesian monoidal, then the 2-category of categories which are fibre-wise monoidally opfibred over $\mathcal{X}$ is equivalent to the 2-category of categories which are globally monoidally opfibred over $\mathcal{X}$.

$$\mathsf{fOpFib}(\mathcal{X}) \cong \mathsf{gOpFib}(\mathcal{X})$$

Network Models

- FP is monoidally opfibred over $\mathbb{N}[C]$

$$\begin{tikzcd} FP \arrow[d] \\ \mathbb{N}[C] \end{tikzcd}$$

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$$\mathbb{N}[C] \to \mathsf{Cat}$$

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- Let S denote the free symmetric monoidal category functor

$$\mathsf{S}[C] \to \mathbb{N}[C]$$

Network Models

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- inverse monoidal Grothendieck construction to get an indexed category

$$\mathbb{N}[C] \to \mathsf{Cat}$$

- Let S denote the free symmetric monoidal category functor

$$\mathsf{S}[C] \to \mathbb{N}[C]$$

- composite is a monoidal indexed category

$$\mathsf{S}[C] \xrightarrow{i} \mathbb{N}[C] \xrightarrow{p} \mathsf{Cat}$$

Theorem (Baez, Foley, M.)

The global monoidal indexed category $G\colon \mathsf{S}(C) \to \mathsf{Cat}$ *lifts to a functor* $\hat{G}\colon \mathsf{S}(C) \to \mathsf{PreMonCat}$*:*

$$\begin{tikzcd} & \mathsf{PreMonCat} \arrow[d, "U"] \\ \mathsf{S}(C) \arrow[ur, "\hat{G}"] \arrow[r, "G"'] & \mathsf{Cat} \end{tikzcd}$$

Network Models

- monoidal functor

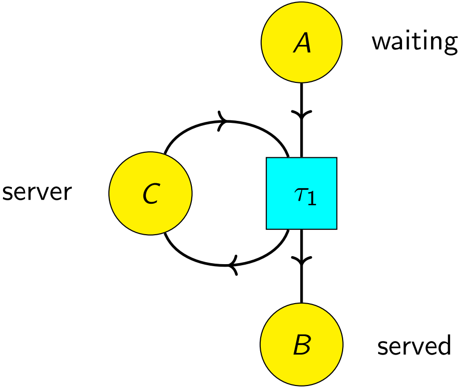
$$\mathsf{S}[C] \xrightarrow{\hat{G}} \mathsf{PreMonCat}$$

- monoidal Grothendieck construction gives a monoidal category
  - objects = same objects as FP, markings
  - morphisms = sequential executions + permutations of catalyst tokens
  - tensor = concurrent execution + permutation sum

this gives a variant of the category FP which models **individual token philosophy** on the catalyst tokens, and **collective token philosophy** on all others

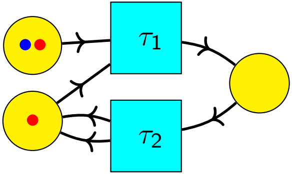
Future

- applications to queueing theory



Future

- applications to queueing theory
- Petri nets with guards



Future

- applications to queueing theory
- Petri nets with guards
- model individual token philosophy by mimicking the usual theory, but over a cocartesian base

John Baez, John Foley, and Joseph Moeller.
Network models from Petri nets with catalysts.
arXiv:1904.03550 [math.CT], 2019.

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Monoidal Grothendieck construction.
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