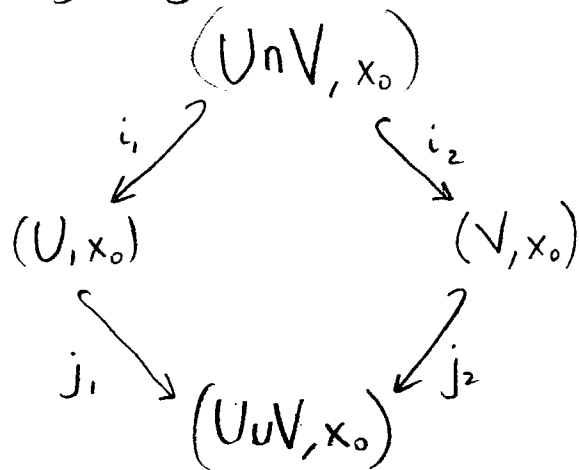


Hw: 1) Show the quotient map $\pi: S^n \rightarrow \mathbb{R}P^n$ is a covering map
(see Thm 60.3)

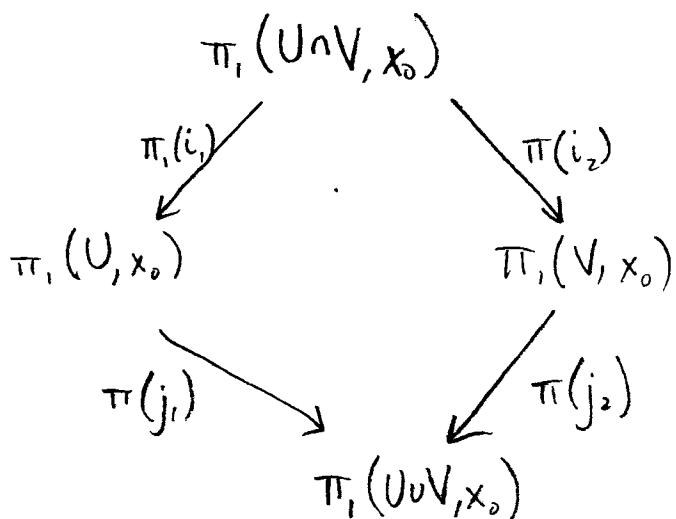
Also 60.2, 3.

Seifert-van Kampen Thm: Assume (X, x_0) is a pointed space,

$U, V \subseteq X$ are open sets containing the basepoint x_0 , $U \cup V = X$, and $U \cap V$ is path-connected. Then applying the functor $\pi_1: \text{Top} \rightarrow \text{Grp}$ to the commuting diagram

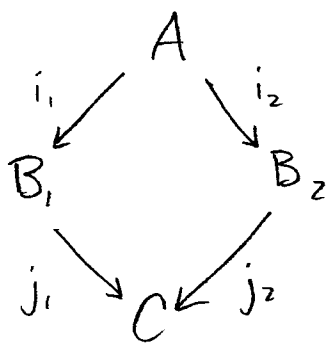


we get a commuting diagram

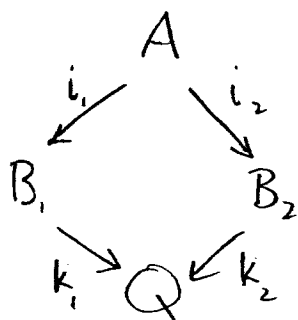


and this diagram is a pushout.

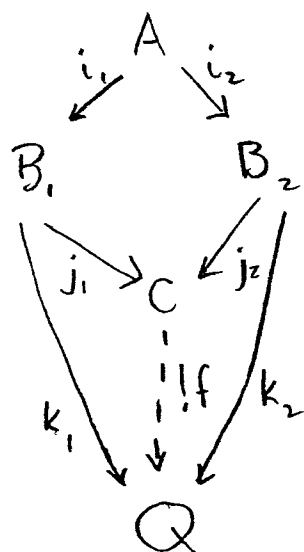
Defn: in any category, we say a commuting diagram



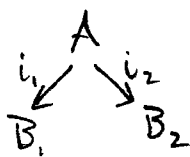
is a pushout if given any other commuting diagram



there exists a unique morphism $f: C \rightarrow Q$ such that



commutes. We often say C is the pushout of



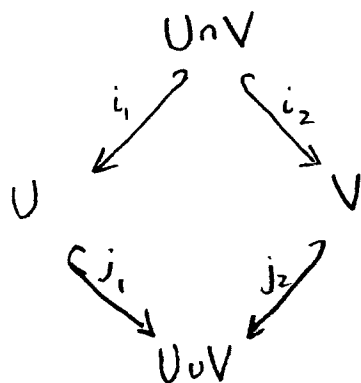
but to be precise we need to specify j_1, j_2 as well.

the universal property

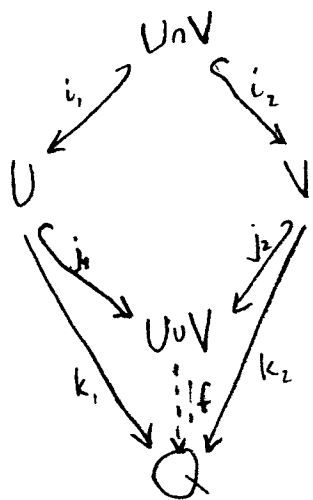
Pushouts look scary, but in fact, they're just a generalization of the concept of the union of sets from the category Set to all other categories.

Here's how "union" is an example of a pushout:

Say U, V are two sets. Then we have a commuting diagram



in Set . We need to show that the universal property holds for $(U \cup V, j_1, j_2)$, i.e. given some other set Q and maps $k_1: U \rightarrow Q$, $k_2: V \rightarrow Q$ there exists a unique $f: U \cup V \rightarrow Q$ such that



commutes,

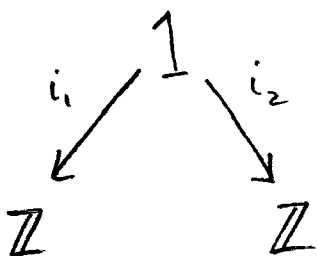
Define $f \circ j_1 = k_1$;
 $f \circ j_2 = k_2$.

To show f exists we have to show that f is well-defined on $U \cup V$. Say $x \in U \cap V$; then $i_1(x) = x \in U$, $i_2(x) = x \in V$. Our definition says $f \circ j_1 \circ i_1 = k_1 \circ i_1 = f \circ j_2 \circ i_2 = k_2 \circ i_2$ which is true because our diagram commutes.

The moral: to define a function $f: U \cup V \rightarrow Q$ we can give two functions $k_1: U \rightarrow Q$, $k_2: V \rightarrow Q$ that agree on $U \cap V$.

Another moral: if you can define a concept without referring to the underlying category, you can apply it anywhere.

Pushout example in Grp: Given



where i_1, i_2 are the unique homomorphism from the trivial group 1 to \mathbb{Z} , the pushout is the free group on two generators

