

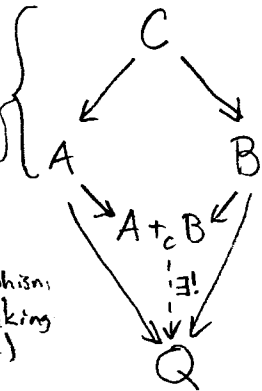
ADVANCED TOPICS

We've talked about pushouts - these are examples of "colimits." There's also an important class of constructions called "limits." Limits & colimits are concepts that make sense in any category. Say we have some category; we can ask if there are objects with some particular universal property.

COLIMITS

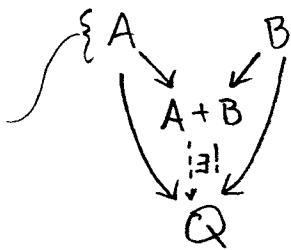
pushout

(note that you can get from anywhere in this part both to $A+B$ and to Q and there's a unique morphism from $A+B$ to Q making the diagram commute.)



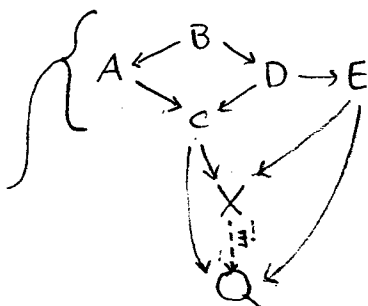
coproduct

ditto for $A+B, Q$



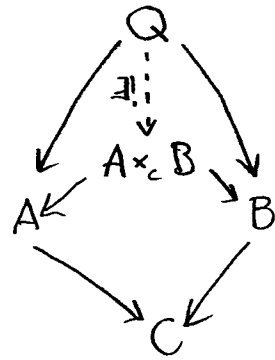
more general colimit

ditto for X, Q



LIMITS

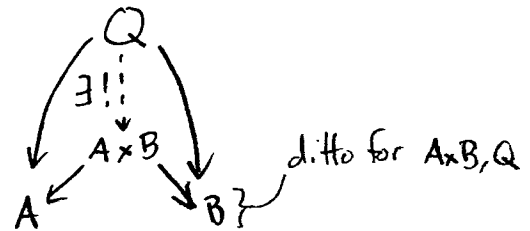
pullback



note that you can get to anywhere in this part

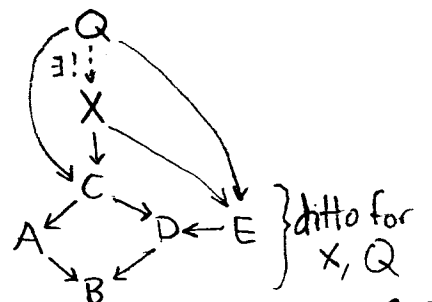
both from $A \times B$ and from Q and there's a unique morphism $Q \rightarrow A \times B$ making the diagram commute.

product



ditto for $A \times B, Q$

more general limit



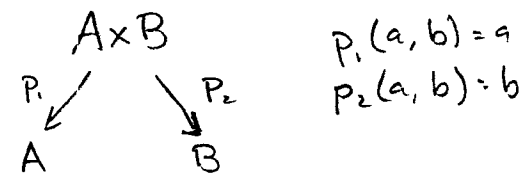
ditto for X, Q

	Set	Top	Top*	Grp	AbGrp
Product $A \times B$	$A \times B$ (cartesian product)	$A \times B$	$(A, a_0) \times (B, b_0)$ $= (A \times B, (a_0, b_0))$	$A \times B$ $= A \oplus B$	$A \oplus B$
Coproduct $A + B$	$A + B =$ $A \cup B$	$A + B =$ $A \cup B$	$(A, a_0) \vee (B, b_0)$ $= \frac{A \cup B}{a_0 \sim b_0}$	$A * B$ "free product"	$A \oplus B$

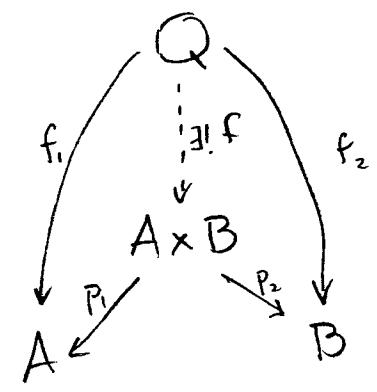


- Set: sets & functions
- Top: spaces & maps
- Top*: pointed spaces & pointed maps
- Grp: groups & group homomorphisms
- AbGrp: abelian groups & group homomorphisms

Given sets A, B , I claim their (categorical) product is the cartesian product:



Given any set Q with functions $f_1: Q \rightarrow A, f_2: Q \rightarrow B$, there exists a unique function f to the product such that this diagram commutes:



$$\begin{aligned}
 p_1(f(q)) &= f_1(q) \\
 p_2(f(q)) &= f_2(q)
 \end{aligned}
 \iff f(q) = (f_1(q), f_2(q))$$

In Set, $A+B$ is the disjoint union of A, B :

"A function $A \cup B \rightarrow Q$ is equivalent to a function $A \rightarrow Q$ and a function $B \rightarrow Q$ "