If $\pi_1(X) \not\cong \pi_1(Y)$ then $X \not\simeq Y$, but

16 Mar 07

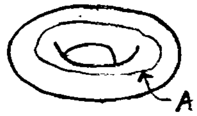
$$\pi_1(X,*) \cong \pi_1(Y,*) \not\Rightarrow X \simeq Y$$

$X = \{*\}$ $\quad \pi_1(X,*) = 1 \quad \pi_n(*,*) = 1$

$Y = S^n$ $\quad \pi_1(Y,*) = 1 \quad \pi_n(S^n,*) = \mathbb{Z}$ $\quad$ but $X \simeq Y$ would imply $\pi_n(X,*) \cong \pi_n(Y,*)$

$n > 1$

$A \subseteq T^2$



Is A a retract of T^2?

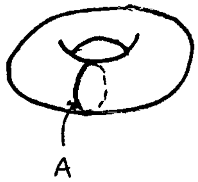
Yes!

$r: S^1 \times S^1 \longrightarrow S^1$
$(x,y) \longmapsto x$

Deformation retract?

No!

$\mathbb{Z} \cong \pi_1(S^1) \not\cong \pi_1(T^2) \cong \mathbb{Z}^2$



retract?

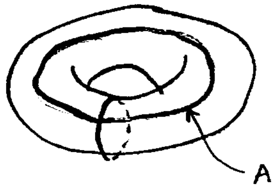
Yes!

$r: S^1 \times S^1 \longrightarrow S^1$
$(x,y) \longmapsto y$

Deformation retract?

No!

Same argument.



retract?

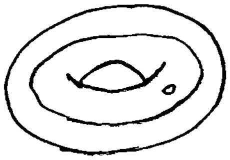
No!

Def. retract?

No!

$\mathbb{Z} * \mathbb{Z} \cong \pi_1(\infty) \not\cong \pi_1(T^2) \cong \mathbb{Z}^2$

$A \subseteq T^2 - *$



retract?

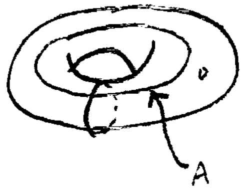
Yes!

restrict the first retraction

Thm: If $A \subseteq B \subseteq X$ and A is a retract of X, then A is a retract of B.

Def. retract?

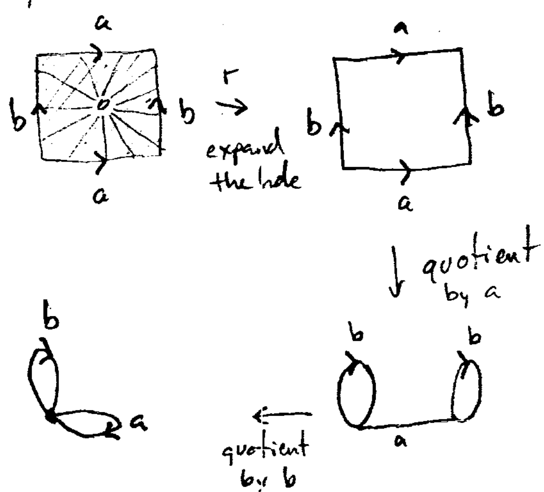
No! There's a deformation retract to the figure 8 which has a different fundamental group ($\pi_1(\infty,*) \cong \mathbb{Z} * \mathbb{Z}$) from $A \cong S^1$ ($\pi_1(S^1,*) \cong \mathbb{Z}$)



$A \subseteq T^2 - \{*\}$

retract?

Yes!



Def. retract?

Yes!

Yes! "Same" as on left.

parameterize the spokes